1) $q_x^{(\tau)} = 1 - p_x^{(\tau)} = 1 - .94 = .06$

$q_x^{(\tau)} = \sum q_x^{(j)} = q_x^{(1)} + q_x^{(2)}$

$\Rightarrow .06 = .04 + q_x^{(2)} \Rightarrow q_x^{(2)} = .02$

2) $p_x^{(\tau)} = \prod p_x'^{(j)}$

$q_x'^{(2)} = .1 \Rightarrow p_x'^{(2)} = .9$

$\therefore p_x^{(\tau)} = p_x'^{(1)} \cdot p_x'^{(2)} = (.95)(.9) = .855$

$\Rightarrow q_x^{(\tau)} = 1 - p_x^{(\tau)} = .145$

3) For (a), we use $q_x^{(2)} = q_x^{(\tau)} - q_x^{(1)}$

$q_x^{(\tau)} = 1 - p_x^{(\tau)} = .28 \Rightarrow q_x^{(2)} = .28 - .09 = .19$

For (b), we use $p_x'^{(2)} = \dfrac{p_x^{(\tau)}}{p_x'^{(1)}} = \dfrac{.72}{.9} = .8$

$\therefore q_x'^{(2)} = 1 - p_x'^{(2)} = .2$

4) $p_{30}^{(\tau)} = \frac{712}{1000} \Rightarrow q_{30}^{(\tau)} = 0.288 = 0.09 + q_{30}^{(2)} \Rightarrow q_{30}^{(2)} = 0.198$

$p_{31}^{(\tau)} = p_{31}'^{(1)} \cdot p_{31}'^{(2)} = 0.68 \Rightarrow q_{31}^{(\tau)} = 0.32 = q_{31}^{(1)} + 0.2 \Rightarrow q_{31}^{(1)} = 0.12$

$l_{32} = 712 \cdot (0.68) = 484.16 \Rightarrow p_{32}^{(\tau)} = \frac{305.0208}{484.16} = 0.63$

$\Rightarrow q_{32}^{(\tau)} = 0.37 = 0.15 + q_{32}^{(2)} \Rightarrow q_{32}^{(2)} = 0.22$

Using $d_x^{(j)} = l_x \cdot q_x^{(j)}$, we have

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
30	1000	90	198
31	712	85.44	142.4
32	484.16	72.624	106.5152

(a) $_2q_{31}^{(2)} = \frac{_2d_{31}^{(2)}}{l_{31}} = \frac{d_{31}^{(2)} + d_{32}^{(2)}}{l_{31}} = 0.3496$

(b) $_{1|2}q_{30}^{(1)} = \frac{_2d_{31}^{(1)}}{l_{30}} = \frac{d_{31}^{(1)} + d_{32}^{(1)}}{l_{30}} = 0.158064$

5) (a) $${}_5q_x^{(\tau)} = \int_0^5 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(\tau)}\, dt$$

$$\mu_{x+t}^{(\tau)} = \sum_j \mu_{x+t}^{(j)} = 0.1 + 0.2 + 0.3 = 0.6 \quad \text{(constant)}$$

$$\therefore \; {}_tp_x^{(\tau)} = e^{-0.6t}$$

$$\Rightarrow {}_5q_x^{(\tau)} = \int_0^5 e^{-0.6t} \cdot (0.6)\, dt = e^{-0.6t}\Big|_5^0 = 1 - e^{-3}$$

(b) $${}_{5|10}q_x^{(2)} = \int_5^{15} {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(2)}\, dt$$

$$= \int_5^{15} e^{-0.6t} \cdot (0.2)\, dt$$

$$= \frac{0.2}{0.6} e^{-0.6t}\Big|_{15}^{5} = \frac{1}{3} \cdot \left(e^{-3} - e^{-9}\right)$$

Note: Since $\mu_{x+t}^{(2)} = \frac{1}{3} \cdot \mu_{x+t}^{(\tau)}$, then

${}_{5|10}q_x^{(2)} = \frac{1}{3} \cdot {}_{5|10}q_x^{(\tau)}$. We could then use

$${}_{5|10}q_x^{(\tau)} = {}_5p_x^{(\tau)} - {}_{15}p_x^{(\tau)} = e^{-0.6(5)} - e^{-0.6(15)}$$

to get ${}_{5|10}q_x^{(2)} = \frac{1}{3}\left(e^{-3} - e^{-9}\right)$ as above.

6) (a) Since $\mu_x^{(\tau)} = .01 + .02 + .03 = .06$ (constant force)

$${}_{10}q_x^{(\tau)} = 1 - {}_{10}p_x^{(\tau)} = 1 - e^{-10(.06)} = 1 - e^{-.6}$$

(b) Since $\mu^{(2)} = \frac{1}{3}\mu^{(\tau)}$, ${}_{10}q_x^{(2)} = \frac{1}{3} \cdot {}_{10}q_x^{(\tau)}$

$$\Rightarrow {}_{10}q_x^{(2)} = \frac{1}{3}(1 - e^{-.6})$$

(c) $${}_{10|10}q_x^{(\tau)} = {}_{10}p_x^{(\tau)} - {}_{20}p_x^{(\tau)}$$

$$= e^{-10(.06)} - e^{-20(.06)} = e^{-.6} - e^{-1.2}$$

(d) Since $\mu^{(1)} = \frac{1}{6}\mu^{(\tau)}$, ${}_{10|10}q_x^{(1)} = \frac{1}{6} \cdot {}_{10|10}q_x^{(\tau)}$

$$\Rightarrow {}_{10|10}q_x^{(1)} = \frac{1}{6}(e^{-.6} - e^{-1.2})$$

(e) Since $\mu^{(\tau)} = .06$, (constant force) the random variable representing the time until departure follows an exponential distribution with mean $\frac{1}{\mu} = \frac{1}{.06}$

$$\therefore \mathring{e}_x^{(\tau)} = \frac{1}{.06}$$

7) (a) $\mu_{x+t}^{(\tau)} = \sum_j \mu_{x+t}^{(j)} = .06 + .06t$

$$\Rightarrow {}_5p_x^{(\tau)} = e^{-\int_0^5 (.06+.06t)\,dt} = e^{-(.06t+.03t^2)\big|_0^5} = e^{-1.05}$$

$$\therefore {}_5q_x^{(\tau)} = 1 - e^{-1.05}$$

Note: It would have been harder, but correct, to use

$${}_5q_x^{(\tau)} = \int_0^5 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(\tau)}\,dt$$

(b) $${}_5q_x^{(3)} = \int_0^5 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(3)}\,dt$$

Since $\mu_{x+t}^{(3)} = \frac{1}{2} \cdot \mu_{x+t}^{(\tau)}$, then

$${}_5q_x^{(3)} = \frac{1}{2} \cdot {}_5q_x^{(\tau)} = \frac{1}{2}\left(1 - e^{-1.05}\right)$$

(c) $$Pr(J=2 \mid T_x = 5) = \frac{\mu_{x+5}^{(2)}}{\mu_{x+5}^{(\tau)}} = \frac{0.12}{0.36} = \frac{1}{3}$$

(d) $$Pr(J=1 \mid T_x < 5) = \frac{{}_5q_x^{(1)}}{{}_5q_x^{(\tau)}}$$

$${}_5q_x^{(1)} = \int_0^5 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(1)}\,dt$$

$${}_5q_x^{(\tau)} = \int_0^5 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(\tau)}\,dt$$

Since $\mu_{x+t}^{(1)} = \frac{1}{6} \cdot \mu_{x+t}^{(\tau)}$, then ${}_5q_x^{(1)} = \frac{1}{6} \cdot {}_5q_x^{(\tau)}$

$$\therefore Pr(J=1 \mid T_x < 5) = \frac{1}{6}$$

8) Use the given information to complete the table.

(i) $q_{50}^{(\tau)} = .1 \implies p_{50}^{(\tau)} = .9 = \dfrac{l_{51}^{(\tau)}}{l_{50}^{(\tau)}} = \dfrac{900}{l_{50}^{(\tau)}} = l_{50}^{(\tau)} = 1000$

$\implies d_{50}^{(2)} = 25$

(ii) ${}_2p_{50}^{(\tau)} = .825 = \dfrac{l_{52}^{(\tau)}}{l_{50}^{(\tau)}} = \dfrac{l_{52}^{(\tau)}}{1000} \implies l_{52}^{(\tau)} = 825$

(iii) $d_{51}^{(1)} = 2 \cdot d_{51}^{(2)} \implies d_{51}^{(\tau)} = 3d_{51}^{(2)}$

$d_{51}^{(\tau)} = l_{51}^{(\tau)} - l_{52}^{(\tau)} = 900 - 825 = 75$

$\implies 75 = 3d_{51}^{(2)} \implies d_{51}^{(2)} = 25$ and $d_{51}^{(1)} = 50$

(iv) ${}_{2|}q_{50}^{(1)} = \dfrac{d_{52}^{(1)}}{l_{50}^{(\tau)}} \implies d_{52}^{(1)} = 1000(.025) = 25$

$\therefore d_{52}^{(\tau)} = d_{52}^{(1)} + d_{52}^{(2)} = 25 + 25 = 50$

$\implies l_{53}^{(\tau)} = l_{52}^{(\tau)} - d_{52}^{(\tau)} = 825 - 50 = 775$

(a) $q_{50}^{(2)} = \dfrac{d_{50}^{(2)}}{l_{50}^{(\tau)}} = \dfrac{25}{1000} = .025$

(b) ${}_2q_{51}^{(\tau)} = 1 - {}_2p_{51}^{(\tau)} = 1 - \dfrac{l_{53}^{(\tau)}}{l_{51}^{(\tau)}} = 1 - \dfrac{775}{900} = \dfrac{125}{900} = \dfrac{5}{36}$

(c) ${}_{1|}q_{51}^{(1)} = \dfrac{d_{52}^{(1)}}{l_{51}^{(\tau)}} = \dfrac{25}{900} = \dfrac{1}{36}$

(d) ${}_{1|2}q_{50}^{(2)} = \dfrac{{}_2d_{51}^{(2)}}{l_{50}^{(\tau)}} = \dfrac{d_{51}^{(2)} + d_{52}^{(2)}}{l_{50}^{(\tau)}} = \dfrac{25+25}{1000} = .05$

9) We seek $d_x^{(2)} = l_x^{(\tau)} \cdot q_x^{(2)} = 1000 \cdot q_x^{(2)}$

$$\left.\begin{array}{l} \mu_x^{(1)} = .1 \\ \mu_x^{(2)} = .3 \\ \mu_x^{(3)} = .5 \end{array}\right\} \Longrightarrow \mu_x^{(\tau)} = .1 + .3 + .5 = .9 \text{ (constant)}$$

$$\Longrightarrow p_x^{(\tau)} = e^{-.9(1)} = e^{-.9}$$

$$\Longrightarrow q_x^{(\tau)} = 1 - e^{-.9}$$

$$\mu_x^{(2)} = \frac{1}{3}\mu_x^{(\tau)} \Longrightarrow q_x^{(2)} = \frac{1}{3} q_x^{(\tau)} = \frac{1}{3}\left(1 - e^{-.9}\right)$$

$$\therefore\ d_x^{(2)} = 1000 \cdot \frac{1}{3}\left(1 - e^{-.9}\right) \approx 197.81$$

10) (a) $\mu_x^{(\tau)} = .05$ (constant force)

$$\Longrightarrow {}_2p_x^{(\tau)} = e^{-.05(2)} = e^{-.1}$$

(b) $\mu^{(1)} = \frac{2}{5}\mu^{(\tau)} \Longrightarrow {}_2q_x^{(1)} = \frac{2}{5}\,{}_2q_x^{(\tau)}$

$${}_2p_x^{(\tau)} = e^{-.1} \Longrightarrow {}_2q_x^{(1)} = \frac{2}{5}(1-e^{-.1})$$

(c) $\mu^{(2)} = \frac{3}{5}\mu^{(\tau)} \Longrightarrow {}_2q_x^{(2)} = \frac{3}{5}\,{}_2q_x^{(\tau)} = \frac{3}{5}(1-e^{-.1})$

Note that ${}_2q_x^{(2)} + {}_2q_x^{(1)} = 1-e^{-.1} = {}_2q_x^{(\tau)}$

(d) ${}_2p_x'^{(1)} = e^{-\int_0^2 \mu_{x+t}^{(1)}\,dt} \underset{=}{\underline{CF}} e^{-.02(2)} = e^{-.04}$

$$\Longrightarrow {}_2q_x'^{(1)} = 1-e^{-.04}$$

(e) ${}_2p_x'^{(2)} = e^{-\int_0^2 \mu_{x+t}^{(2)}\,dt} \underset{=}{\underline{CF}} e^{-.03(2)} = e^{-.06}$

$$\Longrightarrow {}_2q_x'^{(2)} = 1-e^{-.06}$$

Note that ${}_2p_x'^{(1)} \cdot {}_2p_x'^{(2)} = e^{-.04} \cdot e^{-.06} = e^{-.1} = {}_2p_x^{(\tau)}$

Note:

11) $\mu_{x+t}^{(1)} = \frac{1}{80-x-t}$ (DML($\omega=80$))

(a) $${}_{10}q_{50}^{(1)} = \int_0^{10} {}_tp_{50}^{(\tau)} \cdot \mu_{50+t}^{(1)} \cdot dt$$

$$= \int_0^{10} \underbrace{{}_tp_{50}^{\prime(2)}}_{\stackrel{CF}{=} e^{-.1t}} \cdot \underbrace{{}_tp_{50}^{\prime(1)} \cdot \mu_{50+t}^{(1)}}_{\stackrel{DML}{\underset{\omega=80}{=}} \frac{1}{80-50} = \frac{1}{30}} dt$$

$$= \frac{1}{30} \cdot \int_0^{10} e^{-.1t} dt = \frac{1}{30} \cdot \frac{1}{.1} e^{-.1t}\Big|_{10}^{0} = \frac{1}{3}(1-e^{-1})$$

(b) $$\mu_{50}^{(\tau)}(10) = \mu_{50}^{(1)}(10) + \mu_{50}^{(2)}(10)$$

$$= \frac{1}{80-50-10} + 0.1 = 0.15$$

12) (a) $${}_{10}q_{30}^{(1)} = \int_0^{10} {}_tp_{30}^{(\tau)} \cdot \mu_{30+t}^{(1)}\, dt$$

$$= \int_0^{10} \underbrace{{}_tp_{30}^{\prime(2)}}_{\overset{CF}{=} e^{-.05t}} \cdot \underbrace{{}_tp_{30}^{\prime(1)} \cdot \mu_{30+t}^{(1)}}_{\underset{\omega=100}{\overset{DML}{=}} \frac{1}{100-30} = \frac{1}{70}}\, dt$$

$$= \frac{1}{70} \int_0^{10} e^{-.05t}\, dt = \frac{1}{70} \cdot \frac{1}{.05} e^{-.05t} \Big|_{10}^{0} = \frac{2}{7}\left(1 - e^{-0.5}\right)$$

(b) To calculate ${}_{10}q_{30}^{(2)}$ directly would require integration by parts. So instead, let's use

$$ {}_{10}q_{30}^{(2)} = {}_{10}q_{30}^{(\tau)} - \underbrace{{}_{10}q_{30}^{(1)}}_{\text{see part (a)}}$$

$${}_{10}q_{30}^{(\tau)} = 1 - {}_{10}p_{30}^{(\tau)} = 1 - {}_{10}p_{30}^{\prime(1)} \cdot {}_{10}p_{30}^{\prime(2)}$$

$$= 1 - \left(\frac{100-30-10}{100-30}\right) \cdot e^{-10(.05)} = 1 - \frac{6}{7} \cdot e^{-0.5}$$

$$\therefore\ {}_{10}q_{30}^{(2)} = \left(1 - \frac{6}{7} e^{-0.5}\right) - \frac{2}{7}\left(1 - e^{-0.5}\right) = \frac{5}{7} - \frac{4}{7} e^{-0.5}$$

13) (a) $\quad {}_tp_x^{\prime(j)} \overset{\text{MUDD}}{=} \left[{}_tp_x^{(\tau)}\right]^{q_x^{(j)}/q_x^{(\tau)}}$

$$p_x^{\prime(1)} = .9 \text{ and } p_x^{\prime(2)} = .8 \implies p_x^{(\tau)} = .72 \implies q_x^{(\tau)} = .28$$

$$\left.\begin{matrix} t=1 \\ j=1 \end{matrix}\right\} \implies .9 = (.72)^{q_x^{(1)}/.28} \implies q_x^{(1)} = .28\,\frac{\ln(.9)}{\ln(.72)} \approx .089804$$

$$\left.\begin{matrix} t=1 \\ j=2 \end{matrix}\right\} \implies .8 = (.72)^{q_x^{(2)}/.28} \implies q_x^{(2)} = .28\,\frac{\ln(.8)}{\ln(.72)} \approx .190196$$

<u>Note</u>: $q_x^{(1)} + q_x^{(2)} = .28 = q_x^{(\tau)}$

(b) (SUDD)

$$q_x^{(1)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(1)}\,dt = \int_0^1 \underline{{}_tp_x^{\prime(1)}} \cdot \underline{\underline{{}_tp_x^{\prime(2)}}} \cdot \underline{\mu_{x+t}^{(1)}}\,dt$$

$${}_tp_x^{\prime(1)} \cdot \mu_{x+t}^{(1)} \overset{\text{SUDD}}{=} q_x^{\prime(1)} \qquad {}_tp_x^{\prime(2)} \overset{\text{SUDD}}{=} 1 - {}_tq_x^{\prime(2)} = 1 - t \cdot q_x^{\prime(2)}$$

$$\therefore q_x^{(1)} \overset{\text{SUDD}}{=} \int_0^1 \underline{q_x^{\prime(1)}} \cdot \left(\underline{\underline{1 - t\,q_x^{\prime(2)}}}\right) dt = \int_0^1 .1\,(1 - .2t)\,dt$$

$$= .1(1) - .1\left(\tfrac{.2}{2}\right) = .09$$

Similarly,

$$q_x^{(2)} = \int_0^1 {}_tp_x^{(\tau)}\,\mu_{x+t}^{(2)}\,dt = \int_0^1 \underline{\underline{{}_tp_x^{\prime(1)}}} \cdot \underline{{}_tp_x^{\prime(2)} \cdot \mu_{x+t}^{(2)}}\,dt$$

$$\overset{\text{SUDD}}{=} \int_0^1 \underline{q_x^{\prime(2)}} \cdot \left(\underline{\underline{1 - t \cdot q_x^{\prime(1)}}}\right) dt = \int_0^1 .2\,(1 - .1t)\,dt$$

$$= .2\left[1 - \tfrac{.1}{2}\right] = .19$$

<u>Note</u>: $q_x^{(1)} + q_x^{(2)} = .09 + .19 = .28 = q_x^{(\tau)}$

14) $q_x^{(\tau)} = .1 + .2 = .3 \implies p_x^{(\tau)} = .7$

(a) (MUDD) $\quad {}_tp_x'^{(j)} = \left[{}_tp_x^{(\tau)}\right]^{q_x^{(j)}/q_x^{(\tau)}}$

$\left.\begin{matrix} t=1 \\ j=1 \end{matrix}\right\}$ $p_x'^{(1)} = (.7)^{.1/.3} = (.7)^{1/3} \implies q_x'^{(1)} = 1-(.7)^{1/3}$

$\left.\begin{matrix} t=1 \\ j=2 \end{matrix}\right\}$ $p_x'^{(2)} = (.7)^{.2/.3} = (.7)^{2/3} \implies q_x'^{(2)} = 1-(.7)^{2/3}$

<u>Note</u>: $p_x'^{(1)} \cdot p_x'^{(2)} = (.7)^{1/3} \cdot (.7)^{2/3} = .7 = p_x^{(\tau)}$

(b) (SUDD)

$$0.1 = q_x^{(1)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(1)}\,dt = \int_0^1 \underline{{}_tp_x'^{(1)}} \cdot \underline{\underline{{}_tp_x'^{(2)}}} \cdot \underline{\mu_{x+t}^{(1)}}\,dt$$

$$= \int_0^1 \underline{q_x'^{(1)}} \cdot (1 - t \cdot q_x'^{(2)})\,dt$$

$$\therefore \quad 0.1 = q_x^{(1)} = q_x'^{(1)}\left[1 - \frac{q_x'^{(2)}}{2}\right]$$

Likewise $\quad 0.2 = q_x^{(2)} = q_x'^{(2)}\left[1 - \frac{q_x'^{(1)}}{2}\right]$

2 equations
2 unknowns

To simplify appearance, let $q_x'^{(1)} = a$ and $q_x'^{(2)} = b$

$$\therefore \left.\begin{matrix} 0.1 = a\left(1 - \frac{b}{2}\right) \\ 0.2 = b\left(1 - \frac{a}{2}\right) \end{matrix}\right\}$$

Solving for a in the first equation, we get $a = \frac{0.2}{2-b}$, and substituting into the second equation results in a quadratic in b, $b^2 - 2.1b + .4 = 0$.

$\therefore\ b = q_x'^{(2)} \approx .211847 \implies a = q_x'^{(1)} = \frac{0.2}{2-b} \approx .111847$

Note: $p_x'^{(1)} \cdot p_x'^{(2)} = (1 - q_x'^{(1)})(1 - q_x'^{(2)}) = .7 = p_x^{(\tau)}$

15) MUDD: $\quad {}_tp_x'^{(j)} = \left[{}_tp_x^{(\tau)}\right]^{\left(q_x^{(j)}/q_x^{(\tau)}\right)}$

$$p_x^{(\tau)} = p_x'^{(1)} \cdot p_x'^{(2)} = (.9)(.8) = 0.72 \Rightarrow q_x^{(\tau)} = 0.28$$

$\therefore$ using $t=1$ & $j=2$ in the MUDD formula, we get

$$p_x'^{(2)} = \left[p_x^{(\tau)}\right]^{q_x^{(2)}/q_x^{(\tau)}} \Rightarrow 0.8 = (0.72)^{\left(q_x^{(2)}/0.28\right)}$$

$$\Rightarrow q_x^{(2)} = 0.28 \cdot \frac{\ln(0.8)}{\ln(0.72)} = 0.190196\cdots$$

(a) $\quad {}_{0.3}q_x^{(2)} \overset{\text{MUDD}}{=\!=} 0.3 \cdot q_x^{(2)} = 0.057058\cdots$

(b) $\quad {}_{0.5|0.3}q_x^{(2)} \overset{\text{MUDD}}{=\!=} {}_{0.3}q_x \overset{\text{part}}{\underset{(a)}{=\!=}} 0.057058\cdots$

(c) $\quad {}_{0.3}q_{\underbrace{x+0.5}_{\text{problem}}}^{(2)}$:

Note: $\underbrace{{}_{0.5|0.3}q_x^{(2)}}_{\text{See part (b)}} = \underbrace{{}_{0.5}p_x^{(\tau)}} \cdot \overbrace{{}_{0.3}q_{x+0.5}^{(2)}}^{\text{this is what we seek}}$

$$= 1 - {}_{0.5}q_x^{(\tau)} \overset{\text{MUDD}}{=\!=} 1 - 0.5 \cdot q_x^{(\tau)}$$

$$= 1 - 0.5(0.28) = 0.86$$

$$\therefore \; {}_{0.3}q_{x+0.5}^{(2)} = \frac{0.057058\cdots}{0.86} = 0.066347\ldots$$

16) $${}_{1.5}q_{40}^{(1)} = q_{40}^{(1)} + {}_{1|.5}q_{40}^{(1)} = q_{40}^{(1)} + p_{40}^{(\tau)} \cdot {}_{.5}q_{41}^{(1)}$$

$$p_{40}^{(\tau)} = p_{40}^{\prime(1)} \cdot p_{40}^{\prime(2)} = (.9)(.8) = .72$$

$$q_{40}^{(1)} = \int_0^1 {}_tp_{40}^{(\tau)} \cdot \mu_{40+t}^{(1)}\, dt = \int_0^1 \underline{{}_tp_{40}^{\prime(1)}} \cdot \underline{\underline{{}_tp_{40}^{\prime(2)}}} \cdot \underline{\mu_{40+t}^{(1)}}\, dt$$

$$\underline{\underline{\text{SUDD}}} \quad \underline{q_{40}^{\prime(1)}} \int_0^1 (\underline{\underline{1 - t\, q_{40}^{\prime(2)}}})\, dt$$

$$\therefore q_{40}^{(1)} = q_{40}^{\prime(1)}\left[1 - \frac{q_{40}^{\prime(2)}}{2}\right] = .1\left[1 - \frac{.2}{2}\right] = .09$$

$x=41$

$${}_{.5}q_{41}^{(1)} = \int_0^{0.5} {}_tp_{41}^{(\tau)} \mu_{41+t}^{(1)}\, dt = \int_0^{0.5} \underline{{}_tp_x^{\prime(1)}} \cdot \underline{\underline{{}_tp_x^{\prime(2)}}} \cdot \underline{\mu_{x+t}^{(1)}}\, dt$$

$$\underline{\underline{\text{SUDD}}} \quad \int_0^{.5} \underline{q_{41}^{\prime(1)}} (\underline{\underline{1 - t \cdot q_{41}^{\prime(2)}}})\, dt$$

$$= .1 \int_0^{.5} (1 - .2t)\, dt$$

$$= .1\left[0.5 - .1(.5)^2\right] = .0475$$

$$\therefore {}_{1.5}q_x^{(1)} = .09 + .72(.0475) = .1242$$

17) (a) Since (1) is BOY, then $q_x^{(1)} = q_x^{\prime(1)} = 0.1$

(b) There are several ways to proceed to get $q_x^{\prime(2)}$.

Method 1: $q_x^{(2)} = p_x^{\prime(1)} \cdot q_x^{\prime(2)} \Rightarrow 0.2 = 0.9 \cdot q_x^{\prime(2)}$

$$\Longrightarrow q_x^{\prime(2)} = \frac{2}{9}$$

Method 2: Use totals

$$q_x^{(\tau)} = 0.3 \Longrightarrow p_x^{(\tau)} = 0.7 = p_x^{\prime(1)} \cdot p_x^{\prime(2)}$$

$$\therefore \quad 0.7 = 0.9 \cdot p_x^{\prime(2)} \Longrightarrow p_x^{\prime(2)} = \frac{7}{9}$$

$$q_x^{\prime(2)} = \frac{2}{9}$$

~~17) (See Video Solution)~~

~~(a)~~ $\cancel{q_x'^{(1)} = .1}$

~~(b)~~ $\cancel{q_x'^{(2)} = \frac{2}{9}}$

18) Since decrement 3 is EOY, ignore it until the end. We have $q_x'^{(1)} = .2$, $q_x'^{(2)} = .4$, and $\underbrace{q_x'^{(3)} = .6}_{\text{ignoring until the end}}$

Then

(a) $$q_x^{(1)} = \int_0^1 \underbrace{{}_tp_x^{(\tau)}}_{{}_tp_x'^{(1)}\cdot {}_tp_x'^{(2)}} \cdot \mu_{x+t}^{(1)}\,dt \overset{\text{SUDD}}{=} \int_0^1 q_x'^{(1)}(1 - t\cdot q_x'^{(2)})\,dt$$

$$\Longrightarrow q_x^{(1)} = q_x'^{(1)}\left[1 - \frac{q_x'^{(2)}}{2}\right] = .2\left[1 - \frac{.4}{2}\right] = .16$$

(b) As in (a), $q_x^{(2)} \overset{\text{SUDD}}{=} q_x'^{(2)}\left[1 - \frac{q_x'^{(1)}}{2}\right] = .4\left[1 - \frac{.2}{2}\right] = .36$

(c) Method 1: $q_x^{(\tau)} = q_x^{(1)} + q_x^{(2)} + q_x^{(3)} = .16 + .36 + q_x^{(3)} = .52 + q_x^{(3)}$

$$q_x^{(\tau)} = 1 - p_x^{(\tau)} = 1 - p_x'^{(1)} p_x'^{(2)} p_x'^{(3)} = 1 - (.8)(.6)(.4) = .808$$

$$\Longrightarrow .808 = .52 + q_x^{(3)} \Longrightarrow q_x^{(3)} = .288$$

Method 2: In order to depart by decrement 3, one must survive to the end of the year. Therefore

$$q_x^{(3)} = p_x'^{(1)} \cdot p_x'^{(2)} \cdot q_x'^{(3)} = (.8)(.6)(.6) = .288$$

19) (a) Since (1) is MOY, then

$$q_x^{(1)} = {}_{0.5}p_x^{\prime(2)} \cdot q_x^{\prime(1)}$$

$$= (1 - 0.5\, q_x^{\prime(2)}) \cdot q_x^{\prime(1)}$$

$$\therefore q_x^{(1)} = 0.085$$

(b) Easiest to use totals

$$p_x^{(\tau)} = (.9)(.7) = 0.63 \Rightarrow q_x^{(\tau)} = 0.37 = 0.085 + q_x^{(2)}$$

$$\Longrightarrow q_x^{(2)} = 0.285$$

20) Start with (2) since it's discrete.

$$q_x^{(2)} = {}_{.3}p_x'^{(1)} \cdot \left(.25\, q_x'^{(2)}\right) + {}_{.7}p_x'^{(1)} \cdot \left(.75\, q_x'^{(2)}\right)$$

$$= \left(1 - .3\, q_x'^{(1)}\right) \cdot \left(.25\, q_x'^{(2)}\right) + \left(1 - .7\, q_x'^{(1)}\right) \cdot \left(.75\, q_x'^{(2)}\right)$$

$$\therefore\ q_x^{(2)} = 0.352$$

Now use totals.

$$p_x^{(\tau)} = (0.8)(0.6) = 0.48$$

$$\Rightarrow q_x^{(\tau)} = 0.52 = q_x^{(1)} + 0.352$$

$$\Rightarrow q_x^{(1)} = 0.168$$