

(New) MAS2 Exercises (Solutions)

$$1) APV = 5000 \ddot{a}_{35} \stackrel{\text{ILT}}{=} 5000 (15.3926) = 76963$$

$$2) APV = 5000 a_{35}$$

$$a_{35} = \ddot{a}_{35} - 1 \stackrel{\text{ILT}}{=} 14.3926$$

$$\therefore APV = 71963$$

$$3) APV = 500 \cdot {}_{17|} \ddot{a}_{35} = 500 \cdot {}_{17}E_{35} \cdot \ddot{a}_{52}$$

$${}_{17}E_{35} = v^{17} \cdot {}_{17}P_{35} \stackrel{\text{ILT}}{=} (1.06)^{-17} \cdot \frac{l_{52}}{l_{35}} \stackrel{\text{ILT}}{=} 0.3485 \dots$$

$$\ddot{a}_{52} \stackrel{\text{ILT}}{=} 12.8879$$

$$\therefore APV = 2245.749 \dots$$

$$4) APV = 500 \cdot {}_{17|}a_{35} = 500 \cdot {}_{17}E_{35} \cdot a_{52}$$

$${}_{17}E_{35} \stackrel{ILT}{=} 1.3485 \dots \text{ (See \# 3)}$$

$$a_{52} = \ddot{a}_{52} - 1 \stackrel{ILT}{=} 11.8879$$

$$\therefore APV = 2071.497 \dots$$

$$5) APV = 750 \ddot{a}_{32:27} \stackrel{VEP}{=} 750 + 750 v P_{32} \stackrel{ILT}{=} 1456.344 \dots$$

$$6) APV = 8000 \ddot{a}_{35:17|}$$

$$\ddot{a}_{35:17|} = \ddot{a}_{35} - {}_{17}E_{35} \cdot \ddot{a}_{52} \stackrel{ILT}{=} 10.9011 \dots$$

$$\therefore APV = 87208.804 \dots$$

$$7) APV = 8000 a_{35:17|}$$

$$a_{35:17|} = \ddot{a}_{35:17|} - 1 + {}_{17}E_{35}$$

$$\text{OR } a_{35:17|} = (\ddot{a}_{35} - 1) + {}_{17}E_{35} (\ddot{a}_{52} - 1) \left. \vphantom{a_{35:17|}} \right\} \therefore a_{35:17|} \stackrel{ILT}{=} 10.2496 \dots$$

$$\therefore APV = 81996.845 \dots$$

$$8) APV = 2000 \ddot{a}_{35:\overline{17}|}$$

$$\ddot{a}_{35:\overline{17}|} = \ddot{a}_{\overline{17}|} + {}_{17|}\ddot{a}_{35} = \ddot{a}_{\overline{17}|} + \underbrace{{}_{17}E_{35} \cdot \ddot{a}_{52}}_{\text{see \#3}}$$

$$\ddot{a}_{\overline{17}|, 0.06} \stackrel{TVM}{=} 11.1058\dots$$

$$\therefore APV \stackrel{ILT}{=} 31,194.789\dots$$

$$9) APV = 2000 a_{35:\overline{17}|} = 2000 \left(a_{\overline{17}|} + \underbrace{{}_{17}E_{35} \cdot a_{52}}_{\text{see \#4}} \right)$$

$$a_{\overline{17}|, 0.06} \stackrel{TVM}{=} 10.4772\dots$$

$$\therefore APV \stackrel{ILT}{=} 29240.507\dots$$

$$10) APV = 1000 \ddot{a}_{30:40} \stackrel{ILT}{=} 14206.8$$

$$11) APV = 1000 \ddot{a}_{30:40:\overline{10}|}$$

$$\ddot{a}_{30:40:\overline{10}|} \stackrel{ILT}{=} \underbrace{\ddot{a}_{30:40}}_{\stackrel{ILT}{=} 14.2068} - {}_{10}E_{30:40} \cdot \underbrace{\ddot{a}_{40:50}}_{\stackrel{ILT}{=} 12.4784}$$

$${}_{10}E_{30:40} = v^{10} \cdot {}_{10}P_{30:40} = v^{10} \cdot {}_{10}P_{30} \cdot {}_{10}P_{40} \text{ Do directly, OR}$$

$$\text{tricky } (1.06)^{10} \cdot v^{10} \cdot {}_{10}P_{30} \cdot v^{10} \cdot {}_{10}P_{40} = (1.06)^{10} \cdot {}_{10}E_{30} \cdot {}_{10}E_{40}$$

$$\text{Either way, } {}_{10}E_{30:40} \stackrel{ILT}{=} 5260\dots$$

$$\therefore APV \stackrel{ILT}{=} 7643$$

$$12) APV = 500 \ddot{a}_{\overline{30:40}} = 500 (\ddot{a}_{\overline{30}} + \ddot{a}_{\overline{40}} - \ddot{a}_{\overline{30:40}}) \stackrel{ILT}{=} 8232.95$$

$$13) APV = 500 \ddot{a}_{\overline{30:40}:\overline{10}}$$

$$\ddot{a}_{\overline{30:40}:\overline{10}} = \ddot{a}_{\overline{30}:\overline{10}} + \ddot{a}_{\overline{40}:\overline{10}} - \underbrace{\ddot{a}_{\overline{30:40}:\overline{10}}}_{\text{See \#11}}$$

$$\ddot{a}_{x:\overline{10}} = \ddot{a}_x - {}_{10}E_x \cdot \ddot{a}_{x+10}$$

$$\therefore \ddot{a}_{\overline{30}:\overline{10}} \stackrel{ILT}{=} 7.746 \dots$$

$$\ddot{a}_{\overline{40}:\overline{10}} \stackrel{ILT}{=} 7.696 \dots$$

$$\therefore APV \stackrel{ILT}{=} 3900$$

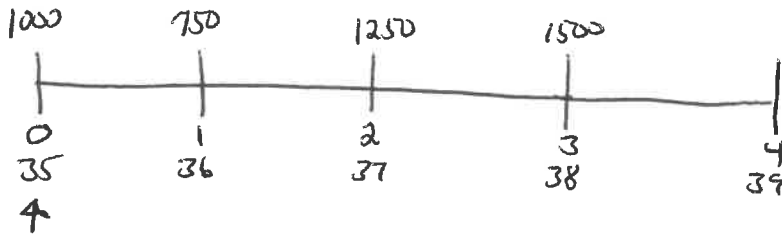
$$14) APV = 1000 \ddot{a}_{\overline{30:40}} + 750 \ddot{a}_{\overline{40:30}} + 500 \ddot{a}_{\overline{30:40}}$$

$$\ddot{a}_{\overline{40:30}} = \ddot{a}_{\overline{40}} - \ddot{a}_{\overline{30:40}}$$

$$\therefore APV = 1000 \ddot{a}_{\overline{30:40}} + 750 (\ddot{a}_{\overline{40}} - \ddot{a}_{\overline{30:40}}) + 500 (\ddot{a}_{\overline{40}} - \ddot{a}_{\overline{30:40}})$$

$$\therefore APV = 750 \ddot{a}_{\overline{30}} + 500 \ddot{a}_{\overline{40}} - 250 \ddot{a}_{\overline{30:40}} \stackrel{ILT}{=} 15748.675$$

15)



$$v = 1 - d = .95$$

$$APV \stackrel{VEP}{=} 1000 + 750 v P_{35} + 1250 v^2 {}_2P_{35} + 1500 v^3 {}_3P_{35}$$

$${}_0P_{35} = .02$$

$${}_2P_{35} = P_{35} \cdot P_{36}$$

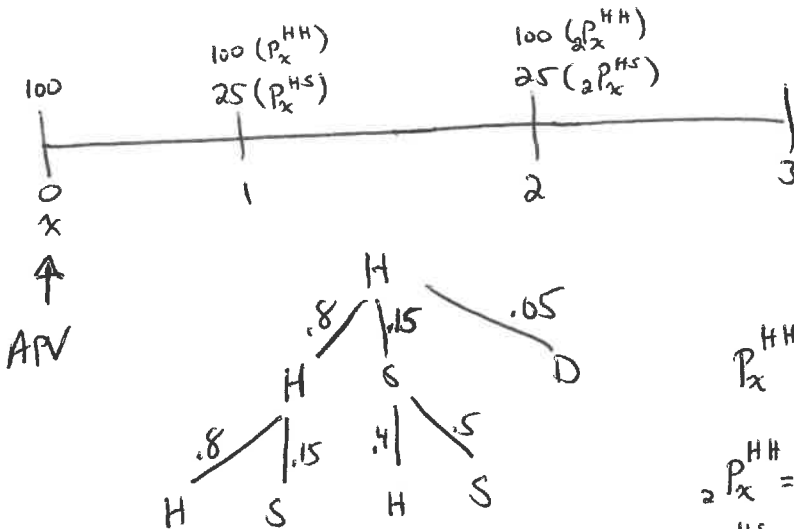
$${}_1P_{36} = .025$$

$${}_3P_{35} = P_{35} \cdot P_{36} \cdot P_{37}$$

$${}_0P_{37} = .03$$

$$\therefore APV = 3968$$

16)



$$v = .95$$

$$P_x^{HH} = .8 \quad P_x^{HS} = .15$$

$${}_2P_x^{HH} = .8^2 + .15(.4) = .7$$

$${}_2P_x^{HS} = .8(.15) + (.15)(.5) = .195$$

$$APV \stackrel{VEP}{=} 100 + 100 v (.8) + 25 v (.15) + 100 v^2 (.7) + 25 v^2 (.195)$$

$$\therefore APV = 247.137...$$

17) See daily notes on 1/18/18

18)

Timeline diagram showing cash flows of 1000 at times 0, 1, 2, ..., 29, 30. The cash flow at time t is $1000(1.05)^t$.

$$APV \stackrel{VEP}{=} 1000 + 1000 \underbrace{(1.05)^1}_{=1} \cdot P_x + 1000 \underbrace{(1.05)^2}_{=1} \cdot {}_2P_x + \dots + 1000 \underbrace{(1.05)^{29}}_{=1} \cdot {}_{29}P_x$$

$$= 1000 (1 + P_x + {}_2P_x + \dots + {}_{29}P_x)$$

Recall, $e_{x:\overline{30}|} = P_x + {}_2P_x + \dots + {}_{30}P_x$

$\therefore 20.7 = P_x + {}_2P_x + \dots + {}_{29}P_x + 0.4$

$\therefore APV = 1000 (1 + (20.7 - 0.4)) = 21,300$

19)

Timeline diagram showing cash flows of 100 at time 0, 200 at time 1, and 300 at time 2.

$$APV = 100 \cdot (I\ddot{a})_x$$

20)

Timeline diagram showing cash flows of 5 at time 0, 7 at time 1, and 9 at time 2.

$APV = 5 \ddot{a}_{25} + 2 (Ia)_{25}$

OR $APV = 3 \ddot{a}_{25} + 2 (I\ddot{a})_{25}$

} 2 "intuitive" answers, other possible correct answers

21)

Timeline diagram showing cash flows of 1500 at time 0, 1400 at time 1, 1300 at time 2, and 600 at time 9.

OR $APV = 1500 \ddot{a}_{50:\overline{10}|} - 100 (Ia)_{50:\overline{9}|}$

OR $APV = 1600 \ddot{a}_{50:\overline{10}|} - 100 (I\ddot{a})_{50:\overline{10}|}$

OR $APV = 500 \ddot{a}_{50:\overline{10}|} + 100 (D\ddot{a})_{50:\overline{10}|}$

} There are other correct answers!

$$22) APV = 4000 \ddot{a}_{40}^{(4)}$$

$$(a) \ddot{a}_{40}^{(4)} \stackrel{UDD}{=} \alpha(4) \cdot \ddot{a}_{40} - \beta(4) \stackrel{ILT}{=} 1.00027 \cdot (14.8166) - .38424$$

$$\therefore APV = 57,745.441 \dots$$

$$(b) \ddot{a}_{40}^{(4)} \stackrel{3WH}{=} \ddot{a}_{40} - \frac{3}{8} - \frac{15}{192} (\mu + \delta) \quad e^{-2\mu} = {}_2P_{39} = \frac{l_{41}}{l_{39}} \Rightarrow \mu = .60269 \dots$$

$$\delta = \ln(1.06) = .058 \dots$$

$$\therefore APV = 57,747.349 \dots$$

$$23) APV = 400 \cdot {}_{20}E_{40} \ddot{a}_{60}^{(4)} = 400 \cdot {}_{20}E_{40} \cdot \ddot{a}_{60}^{(4)}$$

$${}_{20}E_{40} \stackrel{ILT}{=} .27414$$

$$(a) \ddot{a}_{60}^{(4)} \stackrel{UDD}{=} \alpha(4) \cdot \ddot{a}_{60} - \beta(4) \stackrel{ILT}{=} 1.00027 (11.1454) - .38424$$

$$\therefore APV = 1180.355 \dots$$

$$(b) \ddot{a}_{60}^{(4)} \stackrel{3WH}{=} \ddot{a}_{60} - \frac{3}{8} - \frac{15}{192} (\mu + \delta) \quad e^{-2\mu} = {}_2P_{59} = \frac{l_{61}}{l_{59}} \Rightarrow \mu = .0132 \dots$$

$$\delta = \ln(1.06) = .058 \dots$$

$$\therefore APV = 1180.426 \dots$$

$$24) APV = 4000 \ddot{a}_{40:\overline{20}|}^{(4)}$$

$$\ddot{a}_{40:\overline{20}|}^{(4)} = \ddot{a}_{40}^{(4)} - {}_{20}E_{40} \cdot \ddot{a}_{60}^{(4)}$$

$${}_{20}E_{40} \stackrel{ILT}{=} .27414$$

$$\ddot{a}_{40}^{(4)} \stackrel{UDD}{=} \frac{1.00027 (14.8166) - .38424}{1.06} = 14.436 \dots$$

$$\ddot{a}_{60}^{(4)} \stackrel{UDD}{=} \frac{1.00027 (11.1454) - .38424}{1.06} = 10.764 \dots$$

$$\therefore APV = 45941.884 \dots$$

25)

(Rebo 22) $APV = 4000 \ddot{a}_{40}^{(12)}$

(a) $\ddot{a}_{40}^{(12)} \stackrel{UDD}{=} \alpha(12) \ddot{a}_{40} - \beta(12) \stackrel{ILT}{=} 1.00028 (14.8166) - .46812$

$\therefore APV = 57,410.514 \dots$

(b) $\ddot{a}_{40}^{(12)} \stackrel{3WH}{=} \ddot{a}_{40} - \frac{11}{24} - \frac{143}{1728} (\mu + \delta) \quad e^{-2\mu} = {}_2P_{39} = \frac{l_{41}}{l_{39}} \Rightarrow \mu = .00269 \dots$

$\therefore APV = 57,412,887 \dots \quad \delta = \ln(1.06) = .058 \dots$

(Rebo 23) $APV = 400 \cdot {}_{20}E_{40} \ddot{a}_{40}^{(12)} = 400 \cdot {}_{20}E_{40} \ddot{a}_{60}^{(12)}$

${}_{20}E_{40} \stackrel{ILT}{=} .27414$

(a) $\ddot{a}_{60}^{(12)} \stackrel{UDD}{=} \alpha(12) \ddot{a}_{60} - \beta(12) \stackrel{ILT}{=} 1.00028 (11.1454) - .46812$

$\therefore APV = 1171.170 \dots$

(b) $\ddot{a}_{60}^{(12)} \stackrel{3WH}{=} \ddot{a}_{60} - \frac{11}{24} - \frac{143}{1728} (\mu + \delta) \quad e^{-2\mu} = {}_2P_{59} = \frac{l_{61}}{l_{59}} \Rightarrow \mu =$

$\therefore APV = 1171.251 \dots$

(Rebo 24) $APV = 4000 \cdot \ddot{a}_{40:\overline{20}|}^{(12)}$

$\ddot{a}_{40:\overline{20}|}^{(12)} = \ddot{a}_{40}^{(12)} - {}_{20}E_{40} \ddot{a}_{60}^{(12)}$

${}_{20}E_{40} \stackrel{ILT}{=} .27414$

$\ddot{a}_{40}^{(12)} \stackrel{UDD}{=} 1.00028 (14.8166) - .46812$

$\ddot{a}_{60}^{(12)} \stackrel{UDD}{=} 1.00028 (11.1454) - .46812$

$\therefore APV = 45,698.814 \dots$