(New) M2S3 Exercises (Solutions)

1) $\bar{a}_{x:\overline{n}|} = \bar{a}_x - {}_nE_x \cdot \bar{a}_{x+n}$

$= 5 - .55 \cdot (4) = 2.8$

For 2-5) $\mu = .03 \Rightarrow {}_tP_x = e^{-.03t}$

$\delta = .05 \Rightarrow v^t = e^{-.05t}$

2) $APV = 5000\,\bar{a}_x = 5000 \int_0^\infty v^t \cdot {}_tP_x \, dt$

$= 5000 \int_0^\infty e^{-.08t} \, dt = 5000 \cdot \frac{+1}{.08} e^{-.08t} \Big|_\infty^0$

$= \frac{5000}{.08} = 62{,}500$

3) $APV = 1000 \cdot {}_{20|}\bar{a}_x = 1000 \cdot {}_{20}E_x \cdot \bar{a}_{x+n}$

${}_{20}E_x = v^{20} \cdot {}_{20}P_x = e^{-.08(20)} = e^{-1.6}$

$\bar{a}_{x+n} = \int_0^\infty v^t \cdot {}_tP_{x+n} \, dt = \int_0^\infty e^{-.08t} \, dt = \frac{1}{.08} \; (\stackrel{\text{CF}}{=} \bar{a}_x)$

$\therefore APV = 1000\, e^{-1.6} \cdot \frac{1}{.08} = 2523.706\ldots$

4) $APV = 3000\, \bar{a}_{60:\overline{20|}}$

$$\bar{a}_{60:\overline{20|}} = \int_0^{20} v^t \cdot {}_tP_{60}\, dt = \int_0^{20} e^{-.08t}\, dt$$

$$= \frac{+1}{.08} e^{-.08t} \Big|_{20}^{0} = 12.5\left(1 - e^{-1.6}\right)$$

$\therefore APV = 29,928.880\cdots$

5) $APV = 2000 \cdot \bar{a}_{\overline{60:\overline{20|}}}$

$$\bar{a}_{\overline{60:\overline{20|}}} = \bar{a}_{\overline{20|}} + {}_{20}E_{60} \cdot \bar{a}_{80}$$

$$\bar{a}_{\overline{20|}} = \frac{1 - v^{20}}{\delta} = \frac{1 - e^{-1.0}}{.05} = 12.642\cdots$$

$${}_{20}E_{60} = v^{20} \cdot {}_{20}P_{60} = e^{-1.6} = 0.201\cdots$$

$$\bar{a}_{80} = \int_0^{\infty} v^t \cdot {}_tP_{80}\, dt = \int_0^{\infty} e^{-.08t}\, dt = \frac{1}{.08} = 12.5$$

$\therefore APV = 30,332.235\cdots$

6) (a) $\bar{a}_x = \int_0^\infty v^t \cdot {}_tp_x \, dt \underset{\mu_x = .05}{\overset{\delta = .05}{=\!=}} \int_0^\infty e^{-.05t} \cdot e^{-.05t} \, dt$

$= \int_0^\infty e^{-.1t} \, dt = \frac{1}{.1} e^{-.1t} \Big|_\infty^0 = 10$

$\bar{a}_y = \int_0^\infty v^t \cdot {}_tp_y \, dt \underset{\mu_y = .15}{\overset{\delta = .05}{=\!=}} \int_0^\infty e^{-.05t} \cdot e^{-.15t} \, dt$

$= \int_0^\infty e^{-.2t} \, dt = \frac{1}{.2} e^{-.2t} \Big|_\infty^0 = 5$

$\bar{a}_{xy} = \int_0^\infty v^t \cdot {}_tp_{xy} \underset{\substack{\mu_{xy} = \mu_x + \mu_y \\ \text{(independent)}}}{\overset{\delta = .05}{=\!=}} \int_0^\infty e^{-.05t} \cdot e^{.2t} \, dt$

$= \int_0^\infty e^{-.25t} \, dt = \frac{1}{.25} e^{-.25t} \Big|_\infty^0 = 4$

$\bar{a}_{\overline{xy}} = \bar{a}_x + \bar{a}_y - \bar{a}_{xy} = 10 + 5 - 4 = 11$

(b) $APV = 3000 \, \bar{a}_{xy} + 5000 (\bar{a}_x - \bar{a}_{xy}) + 6000 (\bar{a}_y - \bar{a}_{xy})$

$= 5000 \, \bar{a}_x + 6000 \, \bar{a}_y - 8000 \, \bar{a}_{xy}$

$= 5000(10) + 6000(5) - 8000(4)$

$= 48{,}000$

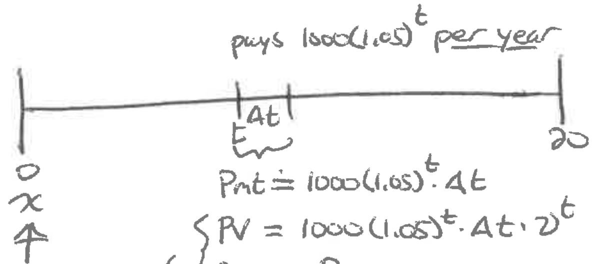
7) $APV = 9\cdot \bar{a}^{00}_{x:\overline{20}|} = 9\cdot\int_0^{20} v^t\cdot {}_tp_x^{00}\,dt$

$$v^t = e^{-.03t} \qquad {}_tp_x^{00} = {}_tp_x^{\overline{00}} = e^{-.06t}$$

$$\therefore APV = 9\cdot\int_0^{20} e^{-.09t}\,dt = 9\cdot\frac{1}{.09}e^{-.09t}\Big|_{20}^{0} = 100(1-e^{-1.8})$$

$$= 83.470\ldots$$

8) APV $\doteq$



$Pmt \doteq 1000(1.05)^t\cdot\Delta t$

$$\begin{cases} PV = 1000(1.05)^t\cdot\Delta t\cdot v^t \\ Pr = {}_tp_x \end{cases} \qquad \Delta t = dt$$

$*\ \to APV = 1000(1.05)^t\cdot v^t\cdot {}_tp_x\cdot dt = \text{integrand}$

$$\therefore APV = \int_0^{20} 1000\,\underbrace{(1.05)^t\cdot v^t}_{i=.05\ 1}\cdot\underbrace{{}_tp_x}_{{}_tp_{40} = \left(\frac{60-t}{60}\right)^3}\,dt \qquad x=40$$

$$= 1000\int_0^{20}\left(\frac{60-t}{60}\right)^3 dt$$

$$= 1000\cdot(+60)\cdot\frac{1}{4}\left(\frac{60-t}{60}\right)^4\Big|_{20}^{0}$$

$$= 15000\cdot\left(1-\left(\frac{40}{60}\right)^4\right) = 12{,}037.037\ldots$$

For 9 & 10 (and 11 also) (and 12 also)

$$\ddot{a}_x^{(m)} \overset{UDD}{=} \alpha(m)\cdot\ddot{a}_x - \beta(m) \xRightarrow{m\to\infty} \bar{a}_x \overset{UDD}{=} \alpha(\infty)\cdot\ddot{a}_x - \beta(\infty)$$

$$\ddot{a}_x^{(m)} \overset{WH}{=} \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\mu_x+\delta) \xRightarrow{m\to\infty} \bar{a}_x \overset{WH}{=} \ddot{a}_x - \frac{1}{2} - \frac{1}{12}(\mu_x+\delta)$$

9) (a) $APV = 1000\,\bar{a}_{40} \overset{UDD}{=} 1000\left(\alpha(\infty)\cdot\ddot{a}_{40} - \beta(\infty)\right) \overset{ILT}{=} 14{,}311$

(b) $APV = 1000\,\bar{a}_{40} \overset{3WH}{=} 1000\left(\ddot{a}_{40} - \frac{1}{2} - \frac{1}{12}(\mu_{40}+\delta)\right)$

μ_{40}: $e^{-2\mu} = {}_2p_{39} = \frac{l_{41}}{l_{39}} \Rightarrow \mu \overset{ILT}{=} .00269\ldots$

$\delta \overset{ILT}{=} \ln(1.06) = .0582\ldots$

$\therefore APV = 14{,}312$

10) (a) $APV = 1000\cdot{}_{20|}\bar{a}_{40} = 1000\cdot{}_{20}E_{40}\cdot\bar{a}_{60} \overset{UDD}{=} 1000\cdot{}_{20}E_{40}\cdot\left(\alpha(\infty)\cdot\ddot{a}_{60} - \beta(\infty)\right)$

$\therefore APV \overset{ILT}{=} 2{,}916$

(b) $APV = 1000\cdot{}_{20|}\bar{a}_{40} = 1000\,{}_{20}E_{40}\cdot\bar{a}_{60} \overset{3WH}{=} 1000\,{}_{20}E_{40}\cdot\left(\ddot{a}_{60} - \frac{1}{2} - \frac{1}{12}(\mu_{60}+\delta)\right)$

μ_{60}: $e^{-2\mu} = {}_2p_{59} = \frac{l_{61}}{l_{59}} \Rightarrow \mu \overset{ILT}{=} .01327\ldots$

$\delta \overset{ILT}{=} \ln(1.06) = .0582\ldots$

$\therefore APV \overset{ILT}{=} 2{,}917$

11) $APV = 1000\,\bar{a}_{40:\overline{20|}} = 1000\left(\bar{a}_{40} - {}_{20}E_{40}\cdot\bar{a}_{60}\right) = [\#9(a)] - [\#10(a)]$

$\therefore APV = 11{,}395$

12) Note: $\bar{a}_x \overset{i=0}{=} \mathring{e}_x$ and $\ddot{a}_x \overset{i=0}{=} 1 + e_x$. Using WH with $i=0$ ($\delta=0$),

we get $\mathring{e}_{50} \overset{WH}{=} (1+e_{50}) - \frac{1}{2} - \frac{1}{12}\cdot\mu_{50}$. For μ_{50}, $e^{-2\mu} = {}_2p_{49} \Rightarrow \mu \overset{ILT}{=} .0057\ldots$

$\therefore \mathring{e}_{50} - e_{50} \overset{3WH}{\underset{ILT}{=}} 0.49952\ldots$