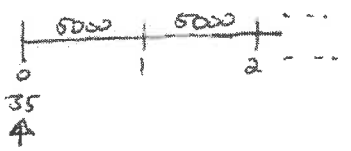
(New) M254 Exercises (Solutions)

1) 

$APV = 5000 A_{35} \overset{ILT}{=} 643.6$

2) 

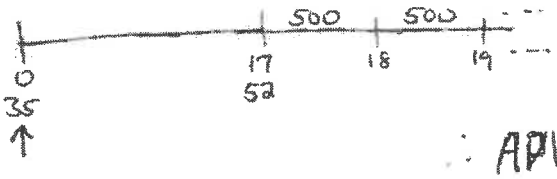
$APV = 350 \cdot A_{40:\overline{10}|} = 350 \cdot {}_{10}E_{40} \overset{ILT}{=} 187.8345$

3) 

$APV = 10000 \cdot A_{35:\overline{17}|} = 10000 \cdot {}_{17}E_{35}$

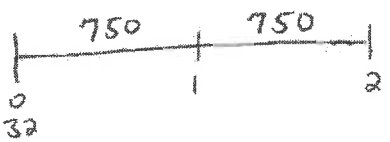
${}_{17}E_{35} = v^{17} \cdot {}_{17}p_{35} = v^{17} \frac{\ell_{52}}{\ell_{35}} \overset{ILT}{=} .3485\cdots$

$\therefore APV \overset{ILT}{=} 3,485$

4) 

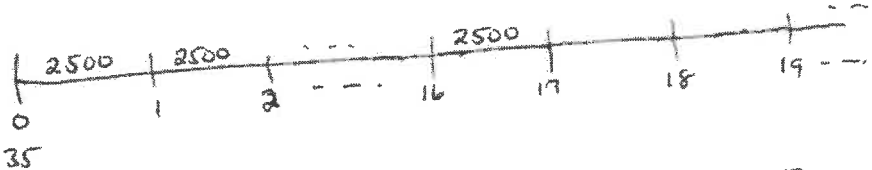
$APV = 500 \cdot {}_{17|}A_{35} = 500 \cdot {}_{17}E_{35} \cdot A_{52}$

$\therefore APV \overset{ILT}{=} 47.135\cdots$

5) 

$APV = 750 \cdot A^{1}_{32:\overline{2}|}$ (Since it's only for 2 years, it's easiest to just VEP)

$\therefore APV = 750 v q_{32} + 750 \cdot v \cdot \underbrace{p_{32} \cdot q_{33}}_{= {}_{1|}q_{32}} \overset{ILT}{=} 2.403\cdots$

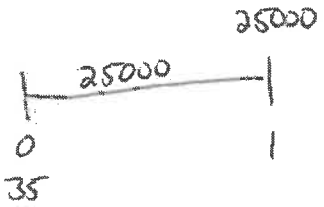
6) 

$APV = 2500 \cdot A^{1}_{35:\overline{17}|} = 2500 \cdot (A_{35} - {}_{17}E_{35} \cdot A_{52})$

$\vdots$

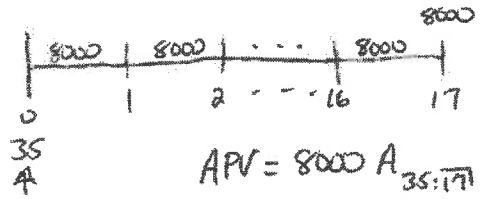
${}_{17}E_{35} \cdot A_{52} \overset{ILT}{=} (\underbrace{.3485\cdots}_{\text{see \#3 above}}) \cdot (.2705)$

$\therefore APV \overset{VEP}{=} 86.123\cdots$

7) 

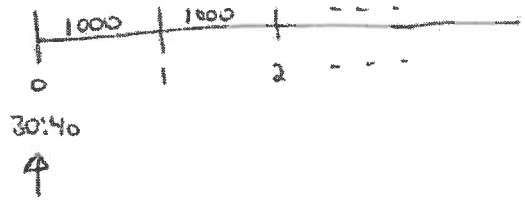
$APV = 25000 \cdot A_{35:\overline{1}|}$ (Since it's only for 1 year, it's easiest to just VEP)

$\therefore APV = 25000v\,{}_1q_x + 25000v\,{}_1p_x = 25000v\,(\underbrace{{}_1q_x + {}_1p_x}_{=1}) \overset{ILT}{=} 23{,}584.905\ldots$

8) 

$APV = 8000\,A_{35:\overline{17}|} = 8000\left(\underbrace{A^{1}_{35:\overline{17}|}}_{\text{see \#6}} + \underbrace{A_{35:\overline{17}|}^{1}}_{={}_{17}E_{35}\ (\text{see \#3})}\right)$

$\therefore APV = 3{,}063.636\ldots$

9) 

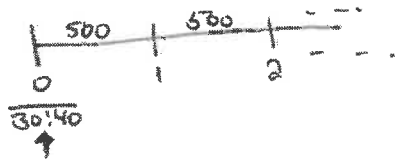
$APV = 1000\,A_{30:40} \overset{ILT}{=} 195.84$

10) 

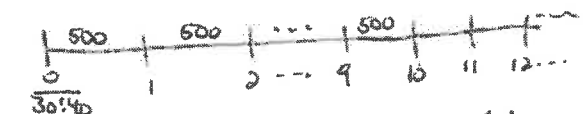
$APV = 1000 \cdot A_{\overline{30:40:\overline{10}|}}^{1} = 1000\left(A_{30:40} - {}_{10}E_{30:40} \cdot A_{40:\overline{30}|}\right)$

${}_{10}E_{30:40} = v^{10} \cdot {}_{10}p_{30} \cdot {}_{10}p_{40} = (1.06)^{-10} \cdot {}_{10}E_{30} \cdot {}_{10}E_{40} \overset{ILT}{=} .5260\ldots$

$\therefore APV \overset{ILT}{=} 41.353\ldots$

11) 

$APV = 500 \cdot A_{\overline{30:40}} = 500\left(A_{30} + A_{40} - A_{30:40}\right) \overset{ILT}{=} 33.98$

12) 

$APV = 500 \cdot A_{\overline{30:40:\overline{10}|}}^{1} = 500\left(A^{1}_{30:\overline{10}|} + A_{40:\overline{10}|}^{1} - A_{30:40:\overline{10}|}^{1}\right) = 0.246\ldots$

13) $APV = 3000 \cdot A_x \overset{VEP}{=} 3000 v\, q_x + 3000 v^2 \,{}_{11}q_x + 3000 v^3 \,{}_{21}q_x + \cdots$

$\mu = -\ln(.9) \Rightarrow p_x = e^{-\mu} = .9 = p \Rightarrow q_x = q = .1$

$\therefore \; {}_{11}q_x = p_x \cdot q_{x+1} = p \cdot q = .9(.1)$

${}_{21}q_x = p^2 \cdot q = (.9)^2(.1), \cdots$

$\therefore APV = 3000 v(.1) + 3000 v^2 (.9)(.1) + 3000 v^3 (.9)^2 (.1) + \cdots$

$\underline{\text{geometric}} \quad \frac{3000 v(.1)}{1 - .9v} \overset{i=.08}{=} 1{,}666.66\cdots$

* Remark: Symbolically, $A_x \overset{CF}{=} \frac{v \cdot q_x}{1 - v \cdot p} \cdot \left(\frac{1+i}{1+i}\right) = \frac{q_x}{1+i-p} = \frac{q_x}{q+i}$

$A_x \overset{CF}{=} \frac{q_x}{q+i}$

14) $APV = 100 A_{x:\overline{12}|} = 100 \cdot {}_{12}E_x = 100 v^{12} \cdot {}_{12}p_x \overset{\text{See \#13}}{\underset{CF}{=}} 100 v^{12} \cdot p^{12}$

$\therefore APV \overset{\substack{i=.08 \\ p=.9}}{=} 11.215\cdots$

15) $APV = 1000 A_{40:\overline{20}|} = 1000 \left(A_{40} - {}_{20}E_{40} \cdot A_{60}\right)$

$A_{40} \overset{CF}{=} \frac{q_x}{q+i} \overset{CF}{=} A_{60} \qquad {}_{20}E_{40} \overset{CF}{=} (vp)^{20}$

$\therefore APV = 1000 \cdot \frac{q_x}{q+i} \left(1 - (vp)^{20}\right) \overset{\substack{i=.08 \\ p=.9}}{=} 541.064\cdots$

16) – 18)

DML($\omega = 110$) mortality $\Rightarrow q_x = {}_{11}q_x = {}_{21}q_x = \cdots = \frac{1}{\omega - x} = \frac{1}{110 - x}$

16) $APV = 3000 \cdot A_{60} \overset{\substack{VEP \\ \omega=110}}{=} 3000 v\, q_{60} + 3000 v^2 \cdot {}_{11}q_{60} + \cdots + 3000 v^{50} \cdot {}_{491}q_{60}$

$\therefore APV = 3000 \cdot \frac{1}{110 - 60} \cdot (v + v^2 + \cdots + v^{50}) = 3000 \cdot \frac{1}{50} \cdot a_{\overline{50}|.08} = 734.009\cdots$

★ Remark: $A_x \overset{DML(\omega)}{=} \frac{1}{\omega - x} \cdot a_{\overline{\omega - x}|}$

17) $APV = 100\,A^{\frac{1}{}}_{50:\overline{12|}} = 100\,{}_{12}E_{50} = 100\,v^{12}\cdot {}_{12}p_{50} = 100(1.08)^{-12}\cdot \frac{110-50-12}{110-50} = 31.769\cdots$

18) $APV = 1000\,A^{1}_{40:\overline{20|}} = 1000\,(A_{40} - \underbrace{{}_{20}E_{40}}_{\text{similar to}}\cdot A_{60})$

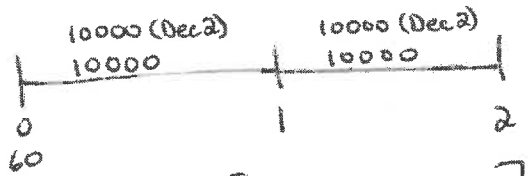
$A_{40} \overset{DML(\omega=110)}{=\!=} \frac{1}{70}\cdot a_{\overline{70|}} \qquad A_{60} \overset{DML(\omega=110)}{=\!=} \frac{1}{50}\cdot a_{\overline{50|}}$

$\therefore APV = 1000\left(\frac{1}{70}a_{\overline{70|}} - (1.08)^{-20}\cdot\frac{5}{7}\cdot\frac{1}{50}a_{\overline{50|}}\right) \overset{i=.08}{=\!=} 140.259\cdots$

★ Remark: Using VEP, we get $A^{1}_{x:\overline{n|}} \overset{DML(\omega)}{=\!=} \frac{1}{\omega-x}\cdot a_{\overline{n|}}$. Alternatively We get,

$\therefore APV = 1000\cdot\frac{1}{70}\cdot a_{\overline{20|}} = 140.259\cdots$

19)



$APV = \left[10000\,A^{1}_{60:\overline{2|}}\right] + \left\{10000\,A^{(Dec\,2)}_{60:\overline{2|}}\right\}$

$\overset{VEP}{=\!=} \left[10000\,v\,q^{(1)}_{60} + 10000\,v^2\cdot {}_{11}q^{(1)}_{60}\right]$

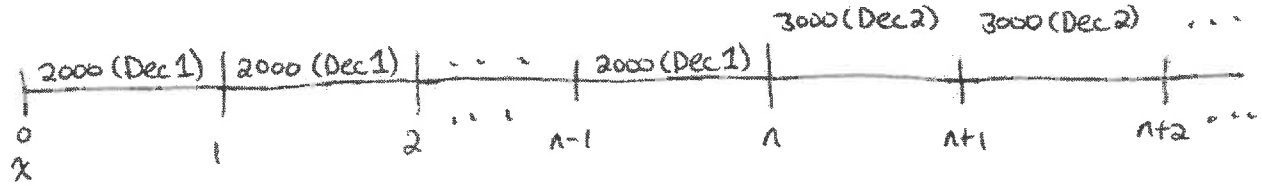
$+ \left\{10000\,v\,q^{(2)}_{60} + 10000\,v^2\cdot {}_{11}q^{(2)}_{60}\right\}$

$q^{(1)}_{60} = .015 \qquad {}_{11}q^{(1)}_{60} = p^{(1)}_{60}\cdot q^{(1)}_{61} = (.985)(.022)$

${}_{11}q^{(2)}_{60} = p^{(1)}_{60}\cdot q^{(2)}_{61} = (.985)(.008)$

$\therefore APV = [339.410\cdots] + \{119.092\cdots\} = 458.503\cdots$

20)



$$APV = 2000\, A^{(Dec\,1)}_{\overline{x:\overline{n}|}} + 3000 \cdot {}_{n|}A^{(Dec\,2)}_{x}$$

$$A^{(Dec\,1)}_{\overline{x:\overline{n}|}} = A^{(Dec\,1)}_{x:\overline{n}|} - {}_{n}E_x = .41763 - .30158 = .11605$$

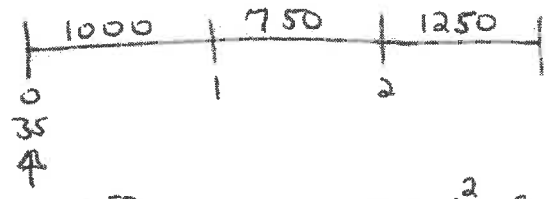
$${}_{n|}A^{(Dec\,2)}_{x} = {}_{n}E_x \cdot A^{(Dec\,2)}_{x+n} = A^{(Dec\,2)}_{x} - A^{(Dec\,2)}_{\overline{x:\overline{n}|}}$$

$$= A^{(Dec\,2)}_{x} - \left(A^{(Dec\,2)}_{x:\overline{n}|} - {}_{n}E_x\right)$$

$$= .46576 - (.58378 - .30158) = .18356$$

$$\therefore APV = 2000(.11605) + 3000(.18356) = 782.78$$

21)



$v = 1 - d = .95$

$$APV \overset{VEP}{=} 1000 v\, {}_{0}q_{35} + 750 v^2 \cdot {}_{11}q_{35} + 1250 v^3 \cdot {}_{21}q_{35}$$

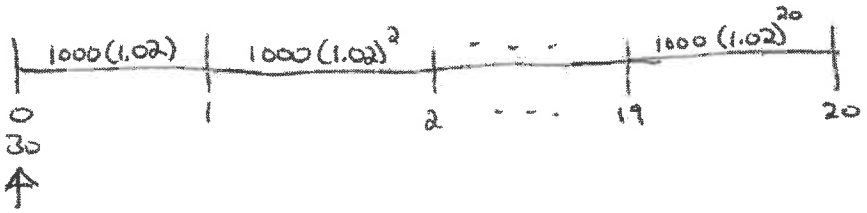
$${}_{0}q_{35} = .02$$

$${}_{11}q_{35} = {}_{1}p_{35} \cdot {}_{1}q_{36} = (.98)(.025)$$

$${}_{21}q_{35} = {}_{2}p_{35} \cdot {}_{1}q_{37} = (.98)(.975)(.03)$$

$$\therefore APV = 66.304\ldots$$

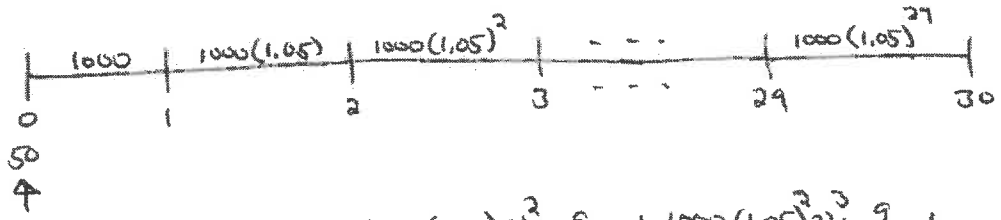
22) $APV^A = \cancel{585} \overset{VEP}{=} 1000 v_{.03} \, {}_{0}q_{30} + 1000 v_{.03}^2 \, {}_{11}q_{30} + \cdots + 1000 v_{.03}^{20} \, {}_{191}q_{30}$

B 

$APV^B \overset{VEP}{=} 1000(1.02) v \, {}_{0}q_{30} + 1000(1.02)^2 \cdot v^2 \, {}_{11}q_{30} + \cdots + 1000(1.02)^{20} \cdot v^{20} \, {}_{191}q_{30}$

If $i = .0506$, then $1.02 v_{.0506} = \frac{1.02}{1.0506} = \frac{1.02}{(1.03)(1.02)} = v_{.03}$

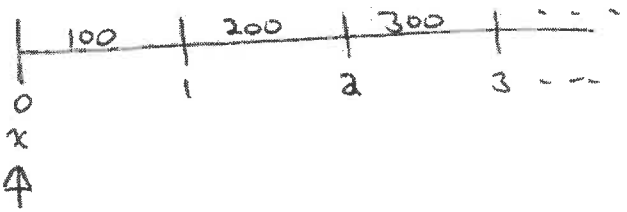
$\therefore APV^B \overset{i=.0506}{=} \cancel{585}$

23) 

$APV = 1000 v \, {}_{0}q_{50} + \underbrace{1000(1.05) v^2}_{i=.05 \; v} \, {}_{11}q_{50} + \underbrace{1000(1.05)^2 v^3}_{i=.05 \; v} \, {}_{21}q_{50} + \cdots + \underbrace{1000(1.05)^{29} \cdot v^{30}}_{i=.05 \; v} \, {}_{291}q_{50}$

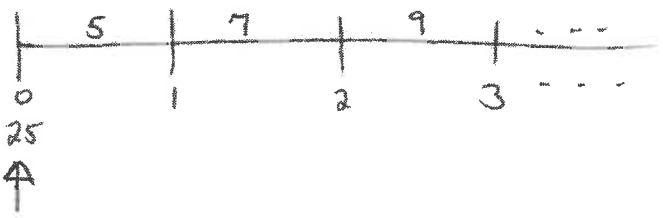
$= 1000 v \left({}_{0}q_{50} + {}_{11}q_{50} + \cdots + {}_{291}q_{50} \right) = 1000 v \cdot {}_{30}q_{50}$

$\therefore APV = \frac{1000}{1.05}(1 - .4) = 571.428\ldots$

24) 

$APV = 100 \cdot (IA)_x$

25)

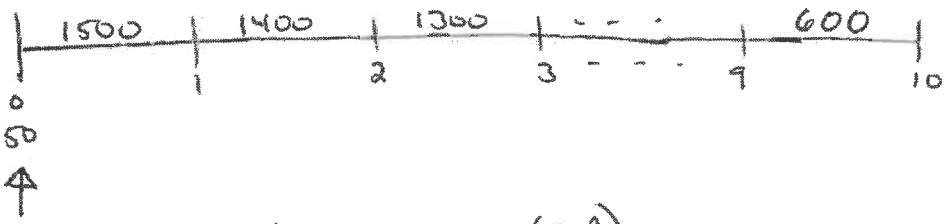


$$APV = 3\,A_{25} + 2\cdot(IA)_{25}$$

OR

$$APV = 5\,A_{25} + 2\cdot(IA)_{26}\cdot {}_1E_{25}$$

26)



$$APV = 500\,A_{\overline{50:\overline{10}|}} + 100\,(DA)_{\overline{50:\overline{10}|}}$$

OR

$$APV = 1600\,A_{\overline{50:\overline{10}|}} - 100\,(IA)_{\overline{50:\overline{10}|}}$$

(There are many other correct answers!)

27) $APV = 1000 \cdot A_{40}^{(4)}$

(a) $APV \overset{UDD}{=} 1000 \cdot \frac{i}{i^{(4)}} \cdot A_{40} \overset{ILT}{=} 1000(1.02223) \cdot (.16132)$
$= 164.906\ldots$

(b) $APV \overset{CAA}{=} 1000(1+i)^{3/8} \cdot A_{40} \overset{ILT}{=} 1000(1.06)^{3/8}(.16132)$
$= 164.883\ldots$

28) $APV = 1000 \cdot A^{(12)}_{\overline{40:20}|}$

(a) $APV = 1000 \cdot \frac{i}{i^{(12)}} \cdot A^{1}_{\overline{40:20}|} = 1000 \frac{i}{i^{(12)}} \cdot \left(A_{40} - {}_{20}E_{40} \cdot A_{60}\right)$

$\therefore APV \overset{ILT}{=} 61.762\ldots$

(b) $APV \overset{CAA}{=} 1000(1+i)^{11/24} \cdot A^{1}_{\overline{40:20}|} \overset{ILT}{=} 1000(1.06)^{11/24} \cdot (A_{40} - {}_{20}E_{40} \cdot A_{60})$

$\therefore APV \overset{ILT}{=} 61.754\ldots$

29) $APV = 1000 \cdot {}_{20|}A_{40}^{(2)} = 1000 \cdot {}_{20}E_{40} \cdot A_{60}^{(2)}$

(a) $APV \overset{UDD}{=} 1000 \cdot {}_{20}E_{40} \cdot \frac{i}{i^{(2)}} \cdot A_{60} \overset{ILT}{=} 102.688\ldots$

(b) $APV \overset{CAA}{=} 1000 \cdot {}_{20}E_{40} \cdot (1+i)^{1/4} \cdot A_{60} \overset{ILT}{=} 102.678\ldots$

30) $APV = 1000\, A_{40:\overline{20|}}^{(4)} = 1000 \cdot \left(A_{40:\overline{20|}}^{1\,(4)} + {}_{20}E_{40} \right)$

(a) $A_{40:\overline{20|}}^{1\,(4)} \overset{UDD}{=} \frac{i}{i^{(4)}} \left(A_{40} - {}_{20}E_{40} \cdot A_{60} \right) \overset{ILT}{=} .06146\ldots$

$\therefore APV = 335.603\ldots$

(b) $A_{40:\overline{20|}}^{1\,(4)} \overset{CAA}{=} (1+i)^{3/8} \cdot \left(A_{40} - {}_{20}E_{40} \cdot A_{60} \right) \overset{ILT}{=} .06145\ldots$

$\therefore APV = 335.594\ldots$