

(New) MAS6 (Part A) Exercises Solutions

$$1) Z = 5000 Z_{35} = 5000 v^{K+1} \quad K = K_{35}$$

$$\begin{aligned} \text{Var}(Z) &= 5000^2 \cdot \text{Var}(Z_{35}) = 5000^2 \cdot \text{Var}(v^{K+1}) \\ &= 5000^2 \cdot ({}^2A_{35} - (A_{35})^2) \stackrel{\text{ILT}}{=} 457,779.04 \end{aligned}$$

$$2) Z = 350 \cdot Z_{40:\overline{1}|} = \begin{cases} 0 & \text{if } K < 10 \\ 350v^{10} & \text{if } K \geq 10 \end{cases}$$

$$\text{Var}(Z) = 350^2 \cdot \text{Var}(Z_{40:\overline{1}|}) = 350^2 \cdot ({}^2A_{40:\overline{1}|} - (A_{40:\overline{1}|})^2)$$

$$A_{40:\overline{1}|} = {}_{10}E_{40} \stackrel{\text{ILT}}{=} .53667$$

$${}^2A_{40:\overline{1}|} = {}^2E_{40} = v^{20} \cdot {}_{10}P_{40} = v^{10} \cdot {}_{10}E_{40} \stackrel{\text{ILT}}{=} .29967 \dots$$

$$\therefore \text{Var}(Z) \stackrel{\text{ILT}}{=} 1428.23 \dots$$

$$3) Z = 10000 Z_{35:\overline{1}|} = \begin{cases} 0 & \text{if } K < 17 \\ 10000v^{17} & \text{if } K \geq 17 \end{cases}$$

$$\text{Var}(Z) = 10000^2 \cdot \text{Var}(Z_{35:\overline{1}|}) = 10000^2 ({}^2A_{35:\overline{1}|} - (A_{35:\overline{1}|})^2)$$

$$A_{35:\overline{1}|} = {}_{17}E_{35} = v^{17} \cdot \frac{l_{52}}{l_{35}} \stackrel{\text{ILT}}{=} .34850 \dots$$

$${}^2A_{35:\overline{1}|} = {}^2E_{35} = v^{34} \cdot \frac{l_{52}}{l_{35}} \stackrel{\text{ILT}}{=} .12942 \dots$$

$$\therefore \text{Var}(Z) \stackrel{\text{ILT}}{=} 796,657.52 \dots$$

$$4) Z = 500 \cdot {}_{171}Z_{35} = \begin{cases} 0 & \text{if } K < 17 \\ 500v^{K+1} & \text{if } K \geq 17 \end{cases}$$

$$\text{Var}(Z) = 500^2 \cdot \text{Var}({}_{171}Z_{35}) = 500^2 \cdot \left({}^2A_{35} - ({}_{171}A_{35})^2 \right)$$

$${}_{171}A_{35} = {}_{17}E_{35} \cdot A_{52} \stackrel{\text{ILT}}{\text{see \#3}} .09427\dots$$

$${}^2A_{35} = {}^2E_{35} \cdot {}^2A_{52} \stackrel{\text{ILT}}{\text{see \#3}} .01396\dots$$

$$\therefore \text{Var}(Z) = 1276.07\dots$$

5) (It's only 2 years!)

$Z = 750Z_{\frac{1}{32}:\overline{2}}$	Pr
$750v$	q_{32}
$750v^2$	${}_{11}q_{32}$
0	${}_2p_{32}$

$$E[Z] = 750v q_{32} + 750v^2 \cdot {}_{11}q_{32} \stackrel{\text{ILT}}{=} 2.395\dots$$

$$E[Z^2] = (750v)^2 \cdot q_{32} + (750v^2)^2 \cdot {}_{11}q_{32} \stackrel{\text{ILT}}{=} 1647.242\dots$$

$$\therefore \text{Var}(Z) = E[Z^2] - (E[Z])^2 \stackrel{\text{ILT}}{=} 1641.503\dots$$

$$6) Z = 2500 \cdot Z_{35:\overline{17}|} = \begin{cases} 2500 v^{K+1} & \text{if } K < 17 \\ 0 & \text{if } K \geq 17 \end{cases}$$

$$\text{Var}(Z) = 2500^2 \cdot \text{Var}(Z_{35:\overline{17}|}) = 2500^2 \cdot ({}^2A_{35:\overline{17}|} - (A_{35:\overline{17}|})^2)$$

$$A_{35:\overline{17}|} = A_{35} - {}_{17}E_{35} \cdot A_{52} \stackrel{\text{ILF}}{=} 0.3444 \dots$$

$${}^2A_{35:\overline{17}|} = {}^2A_{35} - {}_{17}E_{35} \cdot {}^2A_{52} \stackrel{\text{ILF}}{=} 0.02091 \dots$$

$$\therefore \text{Var}(Z) = 123,287.345 \dots$$

$$7) Z = 25000 Z_{35:\overline{17}|} = 25000 v \quad \begin{matrix} \text{(constant,} \\ \text{not random)} \end{matrix}$$

$$\therefore \text{Var}(Z) = 0$$

$$8) Z = 8000 Z_{35:\overline{17}|} = \begin{cases} 8000 v^{K+1} & \text{if } K < 17 \\ 8000 v^{17} & \text{if } K \geq 17 \end{cases}$$

$$\text{Var}(Z) = 8000^2 \cdot \text{Var}(Z_{35:\overline{17}|}) = 8000^2 \cdot ({}^2A_{35:\overline{17}|} - (A_{35:\overline{17}|})^2)$$

$$A_{35:\overline{17}|} = A_{35} - {}_{17}E_{35} \cdot A_{52} + {}_{17}E_{35} \stackrel{\text{ILF}}{=} 0.38295 \dots$$

$${}^2A_{35:\overline{17}|} = {}^2A_{35} - {}_{17}E_{35} \cdot {}^2A_{52} + {}_{17}E_{35} \stackrel{\text{ILF}}{=} 0.15633 \dots$$

$$\therefore \text{Var}(Z) = 235,583.550 \dots$$

$$9) Z = 1000 Z_{30:40} = 1000 v^{K+1} \quad K = K_{30:40}$$

$$\text{Var}(Z) = 1000^2 \cdot \text{Var}(Z_{30:40}) = 1000^2 \cdot ({}^2A_{30:40} - (A_{30:40})^2) \stackrel{\text{ILT}}{=} 283661944..$$

$$10) Z = 1000 \cdot Z_{30:40:\overline{10}|} = \begin{cases} 1000 v^{K+1} & \text{if } K < 10 \\ 0 & \text{if } K \geq 10 \end{cases} \quad K = K_{30:40}$$

$$\text{Var}(Z) = 1000^2 \cdot \text{Var}(Z_{30:40:\overline{10}|}) = 1000^2 \cdot ({}^2A_{30:40:\overline{10}|} - (A_{30:40:\overline{10}|})^2)$$

$$A_{30:40:\overline{10}|} = A_{30:40} - {}_{10}E_{30:40} \cdot A_{40:50} \stackrel{\text{ILT}}{=} .04135..$$

$${}^2A_{30:40:\overline{10}|} = {}^2A_{30:40} - {}_{10}E_{30:40} \cdot {}^2A_{40:50} \stackrel{\text{ILT}}{=} .03035..$$

$$\therefore \text{Var}(Z) = 28,645,394..$$

$$11) Z = \frac{500}{1000} Z_{\overline{30:40}} = \frac{500}{1000} \cdot v^{K+1} \quad K = K_{\overline{30:40}}$$

$$\text{Var}(Z) = 500^2 \cdot \text{Var}(Z_{\overline{30:40}}) = 500^2 \cdot ({}^2A_{\overline{30:40}} - (A_{\overline{30:40}})^2)$$

$$A_{\overline{30:40}} = A_{30} + A_{40} - A_{30:40} \stackrel{\text{ILT}}{=} .06996$$

$${}^2A_{\overline{30:40}} = {}^2A_{30} + {}^2A_{40} - {}^2A_{30:40} \stackrel{\text{ILT}}{=} .00722$$

$$\therefore \text{Var}(Z) = 650,3596$$

$$12) Z = 500 Z_{\overline{30:40:\overline{10}|}} = \begin{cases} 500 v^{K+1} & \text{if } K < 10 \\ 0 & \text{if } K \geq 10 \end{cases}$$

$$\text{Var}(Z) = 500^2 \cdot \text{Var}(Z_{\overline{30:40:\overline{10}|}}) = 500^2 ({}^2A_{\overline{30:40:\overline{10}|}} - (A_{\overline{30:40:\overline{10}|}})^2)$$

$$A_{\overline{30:40:\overline{10}|}} = A_{\overline{30:10}|} + A_{\overline{40:10}|} - A_{\overline{30:40:\overline{10}|}}$$

$${}^2A_{\overline{30:40:\overline{10}|}} = {}^2A_{\overline{30:10}|} + {}^2A_{\overline{40:10}|} - {}^2A_{\overline{30:40:\overline{10}|}}$$

} See above work to determine these values.

$$\therefore \text{Var}(Z) \stackrel{\text{ILT}}{=} 81.134..$$

$$13) Z = 3000 Z_x = 3000 \cdot v^{K+1} \quad K=K_x$$

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$$\text{Var}(Z) = 3000^2 \cdot ({}^2A_x - (A_x)^2)$$

$$i = .08 \Rightarrow \delta = \ln(1.08)$$

$$\mu = -\ln(.9) \Rightarrow p = .9$$

$$A_x \stackrel{\text{CF}}{=} \frac{q}{q+i} = \frac{.1}{.1+.08} \quad \cancel{\frac{.1}{.18}}$$

$${}^2A_x \stackrel{\text{CF}}{=} \frac{q}{q+(i^2+2i)} = \frac{.1}{.1+.1664}$$

$$\therefore \text{Var}(Z) = 600,600.600 \dots$$

$$14) Z = 100 Z_{x:\overline{12}|} = \begin{cases} 0 & \text{if } K < 12 \\ 100v^{12} & \text{if } K \geq 12 \end{cases}$$

$$\therefore \text{Var}(Z) = 100^2 \cdot ({}^2A_{x:\overline{12}|} - (A_{x:\overline{12}|})^2)$$

$$A_{x:\overline{12}|} = {}_{12}E_x \stackrel{\text{CF}}{=} e^{-12(\mu+\delta)} \quad \text{or} \quad v^{12} \cdot {}_{12}P_x = (1.08)^{-12} \cdot (.9)^{12}$$

$${}^2A_{x:\overline{12}|} = {}^2E_x = e^{-12(\mu+2\delta)} \quad \text{or} \quad v^{24} \cdot {}_{12}P_x = (1.08)^{-24} \cdot (.9)^{12}$$

$$\therefore \text{Var}(Z) = 319,598 \dots$$

$$15) Z = 1000 Z_{40:\overline{20}|} = \begin{cases} 1000v^{K+1} & \text{if } K < 20 \\ 0 & \text{if } K \geq 20 \end{cases}$$

$$\text{Var}(Z) = 1000^2 \cdot ({}^2A_{40:\overline{20}|} - (A_{40:\overline{20}|})^2)$$

$$A_{40:\overline{20}|} = A_{40} - {}_{20}E_{40} \cdot A_{60} \stackrel{\text{CF}}{=} \frac{q}{q+i} - e^{-20(\mu+\delta)} \cdot \frac{q}{q+i}$$

$${}^2A_{40:\overline{20}|} = {}^2A_{40} - {}_{20}E_{40} \cdot {}^2A_{60} \stackrel{\text{CF}}{=} \frac{q}{q+(i^2+2i)} - e^{-20(\mu+2\delta)} \cdot \frac{q}{q+(i^2+2i)}$$

$$\therefore \text{Var}(Z) = 80,523,966 \dots$$

$$16) Z = 3000 Z_{60} = 3000 v^{K+1} \quad K = K_{60}$$

$$\text{Var}(Z) = 3000^2 \cdot ({}^2A_{60} - (A_{60})^2)$$

$$A_{60} \stackrel{\text{DML}}{\omega=110} \frac{1}{50} \cdot A_{50:i}$$

$${}^2A_{60} \stackrel{\text{DML}}{\omega=110} \frac{1}{50} \cdot A_{50:i}(i^2+2i)$$

$$\therefore \text{Var}(Z) = 542,469.692 \dots$$

$$17) Z = 100 Z_{50:\frac{1}{12}} = \begin{cases} 0 & \text{if } K < 12 \\ 100v^{12} & \text{if } K \geq 12 \end{cases}$$

$$\text{Var}(Z) = 100^2 \cdot ({}^2A_{50:\frac{1}{12}} - (A_{50:\frac{1}{12}})^2)$$

$$A_{50:\frac{1}{12}} = {}_{12}E_{50} = v^{12} \cdot {}_{12}P_{50} \stackrel{\text{DML}}{\omega=110} v^{12} \cdot \frac{110-50-12}{110-50}$$

$${}^2A_{50:\frac{1}{12}} = {}^2E_{50} = v^{24} \cdot {}_{12}P_{50} \stackrel{\text{DML}}{\omega=110} v^{24} \cdot \frac{110-50-12}{110-50}$$

$$\therefore \text{Var}(Z) = 252,318 \dots$$

$$18) Z = 1000 Z_{40:\overline{20}} = \begin{cases} 1000v^{K+1} & \text{if } K < 20 \\ 0 & \text{if } K \geq 20 \end{cases}$$

$$\text{Var}(Z) = 1000^2 \cdot ({}^2A_{40:\overline{20}} - (A_{40:\overline{20}})^2)$$

$$A_{40:\overline{20}} \stackrel{\text{DML}}{\omega=110} \frac{1}{70} \cdot A_{20:i}$$

$${}^2A_{40:\overline{20}} \stackrel{\text{DML}}{\omega=110} \frac{1}{70} \cdot A_{20:i}(i^2+2i)$$

$$\therefore \text{Var}(Z) = 62,227.160 \dots$$

19) It's only 2 years! (but be careful with multiple decrements)

Z^2	Z	Pr
$(10000v)^2$	$10000v$	$q_{60}^{(1)} = .01$
$(20000v)^2$	$20000v$	$q_{60}^{(2)} = .005$
$(10000v^2)^2$	$10000v^2$	${}_{11}q_{60}^{(1)} = p_{60}^{(1)} \cdot q_{61}^{(1)} = (.985)(.014) = .01379$
$(20000v^2)^2$	$20000v^2$	${}_{11}q_{60}^{(2)} = p_{60}^{(1)} \cdot q_{61}^{(2)} = (.985)(.008) = .00788$
0^2	0	${}_2p_{60}^{(1)} = .96333$

$$v = \frac{1}{1.05}$$

$$E[Z] = 10000v(.01) + 20000v(.005) + 10000v^2(.01379) + 20000v^2(.00788)$$

$$\Rightarrow E[Z] = 458.503 \dots$$

$$E[Z^2] = (10000v)^2(.01) + (20000v)^2(.005) + (10000v^2)^2(.01379) + (20000v^2)^2(.00788)$$

$$\Rightarrow E[Z^2] = 6,448,753.349 \dots$$

$$\therefore \text{Var}(Z) = 6,238,527.98$$

20) It's only 3 years!

Z^2	Z	Pr
$(1000v)^2$	$1000v$	$q_{35} = .02$
$(750v^2)^2$	$750v^2$	${}_{11}q_{35} = (.98)(.025) = .0245$
$(1250v^3)^2$	$1250v^3$	${}_{21}q_{35} = (.98)(.975)(.03) = .028665$
0^2	0	${}_3p_{35} = .926835$

$$v = .95$$

$$E[Z] = 1000(.95)(.02) + 750(.95)^2(.0245) + 1250(.95)^3(.028665)$$

$$E[Z^2] = (1000(.95))^2(.02) + (750(.95)^2)^2(.0245) + (1250(.95)^3)^2(.028665)$$

$$\therefore \text{Var}(Z) = 57802.736 \dots$$