(New) Module 2 Section 6 Exercises (Part B):

The purpose of Numbers 1 through 10 is to verify the consistency of the relationship

$$\ddot{Y}_{\sigma}^{(m)} = \frac{1 - Z_{\sigma}^{(m)}}{d^{(m)}}$$

for the "matching special statuses"

$$\begin{array}{lcl} \sigma = x & \text{and} & \sigma = x{:}\overline{n|} \\ \sigma = xy & \text{and} & \sigma = xy{:}\overline{n|} \\ \sigma = \overline{xy} & \text{and} & \sigma = \overline{xy}{:}\overline{n|} \end{array}$$

Note that by taking expected values on each side, we get $\ddot{a}_{\sigma}^{(m)} = \frac{1-A_{\sigma}^{(m)}}{d^{(m)}}$.
With $m = 1$, this formula gives $\ddot{a}_{\sigma} = \frac{1-A_{\sigma}}{d}$.
With $m = \infty$, this formula gives $\bar{a}_{\sigma} = \frac{1-\bar{A}_{\sigma}}{\delta}$.

In each of Numbers 1 through 10, there are two problems referenced. Review each referenced problem and note that the first is an insurance product on one of the above matching special statuses σ, and the second is an annuity product on that same status using the same actuarial assumptions. We've previously determined the EPV's for these products in the sections in which the problems appear. Now you're to show that the above relationship holds for these EPV's. Note that there may be some round-off error.

1. Number 1 from Section 2 versus Number 1 from Section 4
2. Number 2 from Section 3 versus Number 2(b) from Section 5
3. Number 9(a) from Section 3 versus Number 7(a) from Section 5
4. Number 6 from Section 2 versus Number 8 from Section 4
5. Number 4 from Section 3 versus Number 4(b) from Section 5
6. Number 11 from Section 3 versus Number 8(a) from Section 5
7. Number 10 from Section 2 versus Number 9 from Section 4
8. Number 11 from Section 2 versus Number 10 from Section 4
9. Number 12 from Section 2 versus Number 11 from Section 4
10. Number 13 from Section 2 versus Number 12 from Section 4
11. Number 22(a) from Section 2 versus Number 27(a) from Section 4
12. Number 24 from Section 2 versus Number 30(a) from Section 4

13. The values from Number 1 from Section 3 and Number 1 from Section 5 were attained using $\delta = 0.1$. Show that the numeric values in those problems are consistent with the appropriate formulas above for the $\sigma = x$ and $\sigma = x{:}\overline{n|}$ statuses.

14. Note that the relationship between $\ddot{a}_\sigma$ and A_σ does not hold for statuses other than those listed above. As an example, compare Number 4 from Section 2 to Number 3 from Section 4. Both are *n*-year deferred products and both use the same actuarial assumptions. Show that the EPV's do not satisfy $\ddot{a}_\sigma = \frac{1-A_\sigma}{d}$.

15. Using DML(90) mortality and *i* = .05, determine the variance of the random variable representing the present value of the benefit for a life annuity due issued to (50) with annual payments of 1000.

16. Using ILT actuarial assumptions, determine ${}^2\ddot{a}_{35}$ and $Var(\ddot{Y}_{35})$. Show that $Var(\ddot{Y}_{35}) \neq {}^2\ddot{a}_{35} - (\ddot{a}_{35})^2$, which shows that ${}^2\ddot{a}_{35} \neq E\left[\left(\ddot{Y}_{35}\right)^2\right]$; that is, ${}^2\ddot{a}_{35}$ is NOT equal to the second moment of $\ddot{Y}_{35}$.

17. Using ILT actuarial assumption, determine the variance of the present value random variable for each of the discrete annuities due described:

    (a) whole life issued to (45), with annual payments of 500
    (b) joint life issued to independent lives both age 50, with annual payments of 1000

18. Using ILT actuarial assumption, determine the variance of the present value random variable for a 20-year temporary life annuity due issued to (30) with annual payments of 2000.

19. Given $q_{75} = .02$, $q_{76} = .05$, and *d* = 10% determine the variance of the present value random variable for a 2-year temporary annuity immediate issued to (75) with the first year's payment equal to 15000 and the second year's payment equal to 20000.

20. Using CF($\mu = .03, \delta = .05$) actuarial assumption, determine the variance of the present value random variable for a whole life annuity due issued to (x) with annual payments of 1000.

21. Use ILT actuarial assumptions and the claims acceleration approach to approximate the variance of a whole life annuity due issued to (40) with monthly payments of 250.

22. Use ILT actuarial assumptions and the three-term Woolhouse formula to approximate the variance of a whole life insurance of 10000 issued to (30) with benefit paid at the end of the semiannual period of death.