(New) M2Sb (Part B) Exercises Solutions

1) Show that $\ddot{a}_{35} = \frac{1 - A_{35}}{d}$

2) ——— $\bar{a}_{x} = \frac{1 - \bar{A}_{x}}{\delta}$

3) ——— $\bar{a}_{40} = \frac{1 - \bar{A}_{40}}{\delta}$

4) ——— $\ddot{a}_{35:\overline{17}|} = \frac{1 - A_{35:\overline{17}|}}{d}$

5) ——— $\bar{a}_{60:\overline{20}|} = \frac{1 - \bar{A}_{60:\overline{20}|}}{\delta}$

6) ——— $\bar{a}_{40:\overline{20}|} = \frac{1 - \bar{A}_{40:\overline{20}|}}{\delta}$

7) ——— $\ddot{a}_{30:40} = \frac{1 - A_{30:40}}{d}$

8) ——— $\ddot{a}_{30:40:\overline{10}|} = \frac{1 - A_{30:40:\overline{10}|}}{d}$

9) ——— $\ddot{a}_{\overline{30:40}} = \frac{1 - A_{\overline{30:40}}}{d}$

10) ——— $\ddot{a}_{\overline{30:40}:\overline{10}|} = \frac{1 - A_{\overline{30:40}:\overline{10}|}}{d}$

11) ——— $\ddot{a}^{(4)}_{40} = \frac{1 - A^{(4)}_{40}}{d^{(4)}}$

12) ——— $\ddot{a}^{(4)}_{40:\overline{20}|} = \frac{1 - A^{(4)}_{40:\overline{20}|}}{d^{(4)}}$

13) Show that $\bar{a}_x = \frac{1-\bar{A}_x}{\delta}$ and $\bar{a}_{x:\overline{n}|} = \frac{1-\bar{A}_{x:\overline{n}|}}{\delta}$

14) Show that ${}_{n|}\ddot{a}_{35} \neq \frac{1-{}_{n|}A_{35}}{d}$

15) $Y = 1000\,\ddot{Y}_{50} = 1000 \cdot \frac{1-Z_{50}}{d}$

$\therefore Var(Y) = 1000^2 \cdot \frac{{}^2A_{50} - (A_{50})^2}{d^2}$

$A_{50} \overset{\text{DML}}{\underset{\omega=90}{=}} \frac{1}{40} \cdot a_{\overline{40}|i}$

${}^2A_{50} \overset{\text{DML}}{\underset{\omega=90}{=}} \frac{1}{40} \cdot a_{\overline{40}|(2i+i^2)}$

$Var(Y) = 17,153,571.29$

16) $\ddot{a}_{35} = \frac{1-A_{35}}{d} \Longrightarrow {}^2\ddot{a}_{35} = \frac{1-{}^2A_{35}}{2d-d^2} \overset{\text{ILT}}{=} 8.7735\ldots$

$Var(\ddot{Y}_{35}) = Var\left(\frac{1-Z_{35}}{d}\right) = \frac{{}^2A_{35} - (A_{35})^2}{d^2} = 5.715\ldots$

$Var(\ddot{Y}_{35}) = 5.715\ldots$

${}^2\ddot{a}_{35} - (\ddot{a}_{35})^2 = (8.7735\ldots) - (15.3926)^2 \neq 5.715\ldots$

$\therefore Var(\ddot{Y}_{35}) \neq {}^2\ddot{a}_{35} - (\ddot{a}_{35})^2$

${}^2\ddot{a}_{35} \neq E[(\ddot{Y}_{35})^2]$

17) (a) $Y = 500\,\ddot{Y}_{45} = 500 \cdot \frac{1 - Z_{45}}{d}$

$Var(Y) = 500^2 \cdot \frac{{}^2A_{45} - (A_{45})^2}{d^2} \overset{ILT}{=} 2,148,772.64$

(b) $Y = 1000\,\ddot{Y}_{50:50} = 1000 \cdot \frac{1 - Z_{50:50}}{d}$

$Var(Y) = 1000^2 \cdot \frac{{}^2A_{50:50} - (A_{50:50})^2}{d^2} \overset{ILT}{=} 12,633,184.08$

18) $Y = 2000 \cdot \ddot{Y}_{30:\overline{20}|} = 2000 \cdot \frac{1 - Z_{30:\overline{20}|}}{d}$

$Var(Y) = 2000^2 \cdot \frac{{}^2A_{30:\overline{20}|} - (A_{30:\overline{20}|})^2}{d^2}$

$A_{30:\overline{20}|} = (A_{30} - {}_{20}E_{30} \cdot A_{50}) + {}_{20}E_{30}$

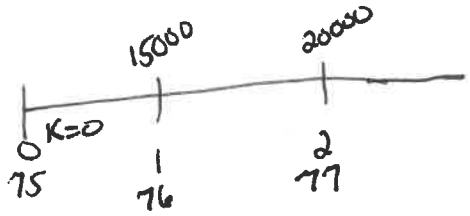
${}^2A_{30:\overline{20}|} = ({}^2A_{30} - {}^2_{20}E_{30} \cdot {}^2A_{50}) + {}^2_{20}E_{30}$

${}^2_{20}E_{30} = v^{40} \cdot {}_{20}P_{30} = v^{20} \cdot {}_{20}E_{30}$

$\therefore Var(Y) \overset{ILT}{=} 4,806,654.9$

19)

K	Y	Pr
0	0	.02
1	15000/2 = 13500	(.98)(.05) = .049
≥ 2	(15000 + 20000)/2 = 29700	0.931



$E[Y] = 13500(.049) + 29700(0.931)$

$E[Y^2] = 13500^2 \cdot (.049) + 29700^2 \cdot (.931)$

$\therefore Var(Y) = 28,575,371.16$

20) $Y = 1000\,\ddot{Y}_x = 1000 \cdot \frac{1 - Z_x}{d}$

$$Var(Y) = 1000^2 \cdot \frac{{}^2A_x - (A_x)^2}{d^2}$$

$$A_x \overset{CF}{=} \frac{q}{q+i}$$

$${}^2A_x \overset{CF}{=} \frac{q}{q + 2i + i^2}$$

$\therefore Var(Y) = 36,014,453.61\ldots$

21) $Y = 3000 \cdot \ddot{Y}^{(12)}_{40} = 3000 \cdot \frac{1 - Z^{(12)}_{40}}{d^{(12)}}$

$$Var(Y) = 3000^2 \cdot \frac{{}^2A^{(12)}_{40} - (A^{(12)}_{40})^2}{(d^{(12)})^2}$$

$$A^{(12)}_{40} \overset{CAA}{=} (1+i)^{11/24} \cdot A_{40}$$

$${}^2A^{(12)}_{40} \overset{CAA}{=} (1+2i+i^2)^{11/24} \cdot {}^2A_{40}$$

$\therefore Var(Y) \overset{ILT}{=}$ 63,512,600.37

22) $Z = 10000\,Z^{(2)}_{30} \Rightarrow Var(Z) = 10000^2 \cdot ({}^2A^{(2)}_{30} - (A^{(2)}_{30})^2)$

$$A^{(2)}_{30} = 1 - d^{(2)} \cdot \ddot{a}^{(2)}_{30} \Rightarrow {}^2A^{(2)}_{30} = 1 - {}^2d^{(2)} \cdot {}^2\ddot{a}^{(2)}_{30}$$

For ${}^2d^{(2)}$: $v = \frac{1}{1+i} = (1 - \frac{d^{(2)}}{2})^2 \Rightarrow v^2 = \frac{1}{(1+i)^2} = (1 - \frac{{}^2d^{(2)}}{2})^2$

$$\Rightarrow {}^2d^{(2)} = 2\left[1 - \frac{1}{1+i}\right]$$

For ${}^2\ddot{a}^{(2)}_{30}$: $\ddot{a}^{(2)}_{30} \overset{3WH}{=} \ddot{a}_{30} - \frac{1}{4} - \frac{1}{48}(\mu+\delta) = \frac{1 - A_{30}}{d} - \frac{1}{4} - \frac{1}{48}(\mu+\delta)$

For μ: $e^{-2\mu} = {}_2P_{29}$

$$\mu = -\tfrac{1}{2} \cdot \ln\left(\frac{l_{31}}{l_{29}}\right)$$

$\mu = .601519\ldots$

$$\Rightarrow {}^2\ddot{a}^{(2)}_{30} \overset{3WH}{=} \frac{1 - {}^2A_{30}}{2d - d^2} - \frac{1}{4} - \frac{1}{48}(\mu + 2\delta)$$

$\therefore Var(Z) \overset{ILT}{=} 1,465,918.424$