MAP 4175 / 5177 Name: KEY

Quiz 10 Date: November 4, 2015

Show all work for full credit, and use correct notation. Simplify answers completely.

1. Given a double decrement model with constant forces $\mu_x^{(1)} = 0.1$ and $\mu_x^{(2)} = 0.2$, determine

(a) $q_x^{(2)}$ $\quad {}_tp_x'^{(1)} = e^{-.1t}$ & ${}_tp_x'^{(2)} = e^{-.2t} \Rightarrow {}_tp_x^{(\tau)} = e^{-.3t}$

$$CF \Rightarrow \frac{q_x^{(2)}}{q_x^{(\tau)}} = \frac{\mu_x^{(2)}}{\mu_x^{(\tau)}} = \frac{.2}{.3} \Longrightarrow q_x^{(2)} = \frac{2}{3}\left(1 - e^{-.3}\right)$$

(b) ${}_{0.3}q_x^{(2)}$ $= \int_0^{0.3} {}_tp_x'^{(1)} \cdot {}_tp_x'^{(2)} \cdot \mu_{x+t}^{(2)}\, dt = \int_0^{0.3} e^{-.3t}(.2)\, dt$

$$= \frac{.2}{.3} e^{-.3t}\Big|_{0.3}^{0} = \frac{2}{3}\left(1 - e^{-.09}\right)$$

2. Given a double decrement model where decrement 1 is BOY and $q_x'^{(1)} = 0.1$, and for decrement 2, $\mu_x^{(2)} = 1.2$, determine $q_x^{(1)}$ and $q_x^{(2)}$.

$$\mu_x^{(2)} = 1.2 \Longrightarrow {}_tp_x'^{(2)} = e^{-1.2t}$$

$$(1)\ \text{BOY} \Longrightarrow q_x^{(1)} = q_x'^{(1)} = .1$$

$$\text{and} \quad q_x^{(2)} = p_x'^{(1)} \cdot q_x'^{(2)} = .9\left(1 - e^{-1.2}\right)$$

3. Given a double decrement model where decrement 1 is EOY and $q_x'^{(1)} = 0.1$, and for decrement 2, $\mu_x^{(2)} = 1.2$, determine $q_x^{(1)}$ and $q_x^{(2)}$.

$$\mu_x^{(2)} = 1.2 \Rightarrow {}_tp_x'^{(2)} = e^{-1.2t}$$

$$(1) \text{ EOY} \Rightarrow q_x^{(2)} = q_x'^{(2)} = 1 - e^{-1.2}$$

$$\text{and} \quad q_x^{(1)} = p_x'^{(2)} \cdot q_x'^{(1)} = e^{-1.2}(.1) = .1e^{-1.2}$$

4. For a four decrement model (whoa!!), decrement 1 is EOY and $q_x'^{(1)} = 0.1$, decrement 2 is BOY and $q_x'^{(2)} = 0.2$, decrement 3 is MOY and $q_x'^{(3)} = 0.3$, and departures from decrement 4 occur continuously throughout the interval from time 0 to time 1. Given ${}_{0.5}p_x'^{(4)} = 0.5$, determine $q_x^{(3)}$.

$$(3) \text{ MOY} \Rightarrow q_x^{(3)} = {}_{.5}p_x'^{(1)} \cdot {}_{.5}p_x'^{(2)} \cdot {}_{.5}p_x'^{(4)} \cdot q_x'^{(3)}$$

$$= (1)(.8)(.5)(.3)$$

$$\Rightarrow q_x^{(3)} = .12$$