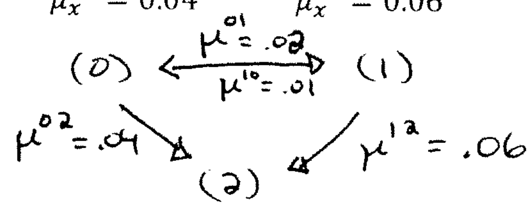
MAP 4175 / 5177 Name:______________________

Quiz 10 Date: November 16, 2016

Show all work for full credit, and use correct notation. You may leave answers in exact form, or you can express the answer as a decimal.

1. For a disability model, where state (0) is the healthy state, state (1) is the disabled state, and state (2) is the dead state, you are given the following non-zero forces of transition:

$\mu_x^{01} = 0.02$ $\mu_x^{10} = 0.01$ $\mu_x^{02} = 0.04$ $\mu_x^{12} = 0.06$



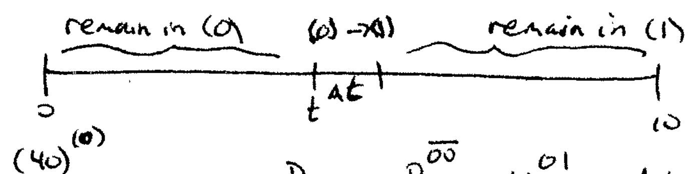
Determine

(a) ${}_{10}p_x^{\overline{00}}$

$$= e^{-\int_0^{10} (\mu^{01} + \mu^{02})\,dt} = e^{-\int_0^{10} .06\,dt} = e^{-0.6}$$

$$= 0.5488\ldots$$

(b) P = the probability that (40), who is healthy, will become disabled exactly once within the next 10 years and remain disabled until age 50.



$$Pr = {}_tp_{40}^{\overline{00}} \cdot \mu_{40+t}^{01} \cdot \Delta t \cdot {}_{10-t}p_{40+t}^{\overline{11}}$$

$$P = \int_0^{10} \underbrace{{}_tp_{40}^{\overline{00}}}_{e^{-.06t}} \cdot \underbrace{\mu_{40+t}^{01}}_{.02} \cdot \underbrace{{}_{10-t}p_{40+t}^{\overline{11}}}_{e^{-.07(10-t)}}\,dt$$

$$= .02e^{-.7} \int_0^{10} e^{.01t}\,dt = \frac{.02e^{-.7}}{.01} e^{.01t}\Big|_0^{10}$$

$$= 2e^{-.7}(e^{.1} - 1) = .10445\ldots$$

2. For a two-life single decrement model, you are given:

State (0) is the state in which both (x) and (y) are alive.
State (1) is the state in which (x) is alive and (y) is dead.
State (2) is the state in which (x) is dead and (y) is alive.
State (3) is the state in which both (x) and (y) are dead.

$$\begin{tikzcd} \substack{x\text{-}A\\y\text{-}A}\ (0) \arrow[r, "\mu_{xy}^{01}=.04"] \arrow[d, "\mu_{xy}^{02}=.04"'] & (1)\ \substack{x\text{-}A\\y\text{-}D} \arrow[d, "\mu_x^{13}=.06"] \\ \substack{x\text{-}D\\y\text{-}A}\ (2) \arrow[r, "\mu_y^{23}=.06"'] & (3)\ \substack{x\text{-}D\\y\text{-}D} \end{tikzcd}$$

The (non-zero) transition rates are:

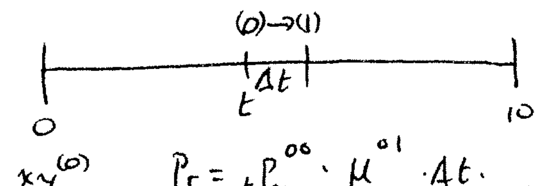
$\mu_{xy}^{01} = \mu_{xy}^{02} = 0.04$ and $\mu_x^{13} = \mu_y^{23} = 0.06$

Determine

(a) ${}_{10}p_{xy}^{00}$ $= {}_{10}P_{xy}^{\overline{00}}$

$$= e^{-\int_0^{10} (.04+.04)\,dt} = e^{-.8} = .4493\cdots$$

(b) ${}_{10}p_{xy}^{01}$ (Note that ${}_{10}p_{xy}^{02} = {}_{10}p_{xy}^{01}$ for this model.)



$$Pr = \underbrace{{}_tP_{xy}^{00}}_{= {}_tP_{xy}^{\overline{00}} = e^{-.08t}} \cdot \mu_{xy}^{01} \cdot \Delta t \cdot \underbrace{{}_{10-t}P_x^{11}}_{= {}_{10-t}P_x^{\overline{11}} = e^{-.06(10-t)}}$$

$\therefore$

$${}_{10}P_{xy}^{01} = \int_0^{10} e^{-.08t} \cdot (.04) \cdot e^{-.06(10-t)}\,dt$$

$$= .04\,e^{-.6} \int_0^{10} e^{-.02t}\,dt$$

$$= \frac{.04e^{-.6}}{.02}\, e^{-.02t} \Big|_{10}^{0} = 2e^{-.6}\left(1-e^{-.2}\right) = .1989\cdots$$

(c) ${}_{10}p_{xy}^{03}$

$$= 1 - {}_{10}P_{xy}^{00} - {}_{10}P_{xy}^{01} - {}_{10}P_{xy}^{02}$$

$$= 1 - {}_{10}P_{xy}^{00} - 2\left({}_{10}P_{xy}^{01}\right) = 1 - .4493\cdots - 2(.1989\cdots)$$

$$= .1527\cdots$$