MAP 4175 / 5177 Name:____________________

Quiz 10 – Take Home Date: November 15, 2017 – Due on November 16, 2017

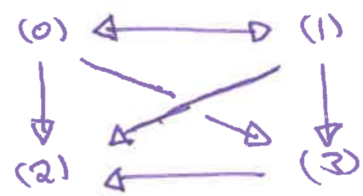
Show all work for full credit, and use correct notation. Simplify answers completely.

A 4-state model has states: Healthy (0), Sick (1), Dead (2), and Terminally Ill (3).

The transition intensities are:

$\mu_x^{01} = 0.0001e^{.06x}$ $\quad \mu_x^{02} = \mu_x^{12} = 6\mu_x^{01}$ $\quad \mu_x^{03} = \mu_x^{13} = 0.05\mu_x^{01}$ $\quad \mu_x^{10} = 0.1\mu_x^{01}$

$\mu_x^{32} = 1.2\mu_x^{02}$



1. Determine ${}_{10}p_{30}^{31}$.

$${}_{10}P_{30}^{31} = 0$$

2. Determine an expression for $\mu_{30+t}^{0\tau}$.

$$\mu_{30+t}^{0\tau} = \mu_{30+t}^{01} + \mu_{30+t}^{02} + \mu_{30+t}^{03} \qquad \frac{\mu_x^{02} = 6\cdot\mu_x^{01}}{\mu_x^{03} = .05\,\mu_x^{01}} \qquad 7.05\cdot\mu_{30+t}^{01}$$

$$\therefore \mu_{30+t}^{0\tau} = 7.05\cdot(.0001e^{.06(30+t)}) = .000705\,e^{1.8}\cdot e^{.06t}$$

3. Determine ${}_{10}p_{30}^{\overline{00}}$.

$${}_{10}P_{30}^{\overline{00}} = e^{-\int_0^{10}\mu_{30+t}^{0\tau}\,dt} = e^{-\int_0^{10}.000705e^{1.8}e^{.06t}\,dt}$$

$$= e^{-.000705e^{1.8}\cdot\frac{1}{.06}e^{.06t}\Big|_0^{10}}$$

$$= e^{-.000705e^{1.8}\cdot\frac{1}{.06}(e^{.6}-1)}$$

$$\therefore {}_{10}P_{30}^{\overline{00}} = 0.9432\ldots$$

4. Given ${}_{10}p_{30}^{00} = 0.943363$ and ${}_{10}p_{30}^{01} = 0.007833$, determine ${}_{10}\dot{p}_{30}^{00}$.

$${}_t\dot{P}_{30}^{00} = {}_tP_{30}^{01} \cdot \mu_{30+t}^{10} - {}_tP_{30}^{00} \cdot \mu_{30+t}^{0\tau}$$

$$\therefore\ {}_{10}\dot{P}_{30}^{00} = (.007833) \cdot \mu_{40}^{10} - (.943363) \cdot \mu_{40}^{0\tau}$$

$$\mu_{40}^{10} = 0.1 \cdot (.0001\, e^{.06(40)}) = .00011\ldots$$

$$\mu_{40}^{0\tau} = .000705\, e^{1.8} \cdot e^{.06(10)} = .00777\ldots$$

$$\therefore\ {}_{10}\dot{P}_{30}^{00} = -0.00733\ldots$$

5. Use Euler's Forward Equation with a step size of 0.125 to approximate the value of ${}_{10.125}p_{30}^{00}$.

$$EM: \quad y(t+h) = y(t) + h \cdot \dot{y}(t)$$

$$y(t) = {}_tP_{30}^{00}$$

$$\left.\begin{array}{l} t = 10 \\ h = 0.125 \end{array}\right\} \Rightarrow {}_{10.125}P_{30}^{00} = {}_{10}P_{30}^{00} + .125 \cdot {}_{10}\dot{P}_{30}^{00}$$

$$= .943363 + .125(-0.00733\ldots)$$

$$\therefore\ {}_{10.125}P_{30}^{00} = 0.9424\ldots$$