Show all work for full credit, and use correct notation. Simplify answers completely.

A 4-state model has states: Healthy (0), Sick (1), Dead (2), and Terminally Ill (3).

The transition intensities are:

\[ \mu_x^{01} = 0.0001e^{0.06x} \quad \mu_x^{02} = \mu_x^{12} = 6\mu_x^{01} \quad \mu_x^{03} = \mu_x^{13} = 0.05\mu_x^{01} \quad \mu_x^{10} = 0.1\mu_x^{01} \]
\[ \mu_x^{32} = 1.2\mu_x^{02} \]

1. Determine \( 10P_{30}^{31} \).

\[ 10P_{30}^{31} = 0 \]

2. Determine an expression for \( \mu_{30+t}^{0r} \).

\[ \mu_{30+t}^{0r} = \mu_{30+t}^{01} + \mu_{30+t}^{02} + \mu_{30+t}^{03} = \frac{\mu_x^{02} \cdot 6\mu_x^{01}}{\mu_x^{03} \cdot 0.05\mu_x^{01}} = 7.05 \cdot \mu_x^{30+t} \]

\[ \therefore \mu_{30+t}^{0r} = 7.05 \cdot (0.0001e^{0.06(30+t)}) = 0.000705e^{1.8} \cdot e^{0.06t} \]

3. Determine \( 10P_{30}^{30} \).

\[ 10P_{30}^{30} = e^{\int_0^{10} \mu_{30+t}^{0r} dt} = e^{\int_0^{10} 0.000705e^{1.8} \cdot e^{0.06t} dt} \]

\[ = e^{-0.000705e^{1.8} \cdot \frac{1}{0.06} e^{0.06t}|_0^{10}} \]

\[ = e^{-0.000705e^{1.8} \cdot \frac{1}{0.06} (e^{0.6} - 1)} \]

\[ \therefore 10P_{30}^{30} = 0.943 \]
4. Given $P_{30}^{00} = 0.943363$ and $P_{30}^{01} = 0.007833$, determine $P_{30}^{00}$.

\[
\begin{align*}
\dot{P}_{30}^{00} &= P_{30}^{01} \cdot \mu_{30}^{10} - P_{30}^{00} \cdot \mu_{30}^{0c} \\
\therefore P_{30}^{00} &= P_{30}^{00} - (0.943363) \cdot \mu_{40}^{10} - (0.007833) \cdot \mu_{40}^{0c} \\
\mu_{40}^{10} &= 0.1 \cdot (0.0001 e^{-0.06(40)}) = 0.00011 \ldots \\
\mu_{40}^{0c} &= 0.00705 e^{1.8} \cdot e^{-0.06(10)} = 0.00777 \ldots \\
\therefore P_{30}^{00} &= -0.00733 \ldots
\end{align*}
\]

5. Use Euler's Forward Equation with a step size of 0.125 to approximate the value of $P_{30}^{00}$.

\[
\begin{align*}
\text{EM: } \gamma(t+h) &= \gamma(t) + h \cdot \dot{\gamma}(t) \\
\gamma(t) &= t \cdot P_{30}^{00} \\
t = 10, \quad h = 0.125 \\
\therefore 10_{125} P_{30}^{00} &= 10 P_{30}^{00} + 0.125 \cdot 10 P_{30}^{00} \\
&= 0.943363 + 0.125(-0.00733 \ldots) \\
\therefore 10_{125} P_{30}^{00} &= 0.9424 \ldots
\end{align*}
\]