MAP 4175 / 5177 Name: KEY

Test 10 Date: November 7, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. (10 points) For a double decrement model, $q_x'^{(1)} = 0.4$, $q_x'^{(2)} = 0.2$, and $q_x^{(1)} = 0.32$. Determine $q_x^{(2)}$.

$p_x'^{(1)} = .6 \quad p_x'^{(2)} = .8 \Rightarrow p_x^{(\tau)} = .48$

$\therefore q_x^{(\tau)} = .52 = q_x^{(1)} + q_x^{(2)} = .32 + q_x^{(2)}$

$\Rightarrow q_x^{(2)} = 0.2$

2. You are given the double decrement table:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$q_x^{(1)}$	$d_x^{(2)}$	$q_x^{(2)}$
95	-	-	0.20	400	-
96	-	400	-	200	0.25
97	-	-	-	-	-

Determine

(a) (5 points) $q_{96}^{(1)} = \frac{d_{96}^{(1)}}{l_{96}} = \frac{400}{l_{96}} \qquad q_{96}^{(2)} = \frac{d_{96}^{(2)}}{l_{96}} \Rightarrow .25 = \frac{200}{l_{96}}$

$\therefore l_{96} = 800$

$\therefore q_{96}^{(1)} = \frac{400}{800} = .5$

(b) (5 points) ${}_2q_{95}^{(\tau)} = 1 - {}_2p_{95}^{(\tau)} = 1 - \frac{l_{97}}{l_{95}}$

$l_{97} = l_{96} - d_{96}^{(1)} - d_{96}^{(2)} = 800 - 400 - 200 = 200$

$l_{95} - d_{95}^{(1)} - d_{95}^{(2)} = l_{96} \Rightarrow l_{95} - .2l_{95} - 400 = 800$

$\Rightarrow l_{95} = 1500$

$\therefore {}_2q_{95}^{(\tau)} = 1 - \frac{200}{1500} = \frac{13}{15}$

3. (5 points each) Use the L-TAM Standard Service Table (SST) to determine

(a) $q_{50}^{(w)} = \frac{d_{50}^{(w)}}{l_{50}} \overset{SST}{=} 0.01977\ldots$

(b) ${}_2q_{45}^{(i)} = \frac{2d_{45}^{(i)}}{l_{45}} = \frac{d_{45}^{(i)} + d_{46}^{(i)}}{l_{45}} \overset{SST}{=} 0.00195\ldots$

(c) ${}_2p_{59}^{(\tau)} = \frac{l_{61}}{l_{59}} \overset{SST}{=} 0.61561\ldots$

(d) ${}_{12|2}q_{50}^{(r)} = \frac{2d_{62}^{(r)}}{l_{50}} = \frac{d_{62}^{(r)} + d_{63}^{(r)}}{l_{50}} \overset{SST}{=} 0.08137\ldots$

4. (10 points) For a triple decrement model, $\mu_x^{(j)}(t) = 0.15 \cdot (4 - j)$. $j = 1, 2, 3$
Determine ${}_{0.1|0.4}q_x^{(2)}$.

$\mu^{(1)} = 0.45 \quad \mu^{(2)} = 0.30 \quad \mu^{(3)} = 0.15$

$\mu^{(\tau)} = .45 + .3 + .15 = 0.90$

${}_{.1|.4}q_x^{(2)} \overset{UF}{=} \frac{\mu^{(2)}}{\mu^{(\tau)}} \cdot {}_{.1|.4}q_x^{(\tau)}$

$= \frac{.3}{.9} \cdot \left[{}_{.1}p_x^{(\tau)} - {}_{.5}p_x^{(\tau)} \right]$

$= \frac{1}{3} \left[e^{-.9(.1)} - e^{-.9(.5)} \right]$

$\therefore \quad {}_{.1|.4}q_x^{(2)} = \frac{1}{3}\left(e^{-.09} - e^{-.45} \right)$

Standard Sickness-Death Model

Functions at $i = 0.05$

Healthy (State 0) can transition to Sick (State 1) or Death (State 2)

Sick (State 1) can transition to Healthy (State 0) or Death (State 2)

Death (State 2) cannot transition

x	$\bar{a}_x^{00}$	$\bar{a}_x^{01}$	$\bar{a}_x^{11}$	$\bar{a}_x^{10}$	$\bar{A}_x^{01}$	$\bar{A}_x^{02}$	$\bar{A}_x^{10}$	$\bar{A}_x^{12}$	${}_{10}p_x^{00}$	${}_{10}p_x^{01}$	${}_{10}p_x^{11}$	${}_{10}p_x^{10}$
50	11.7454	1.9621	12.3919	0.6675	0.24144	0.33126	0.06550	0.36288	0.83936	0.06554	0.81210	0.06063
51	11.4326	2.0306	12.2393	0.5626	0.25196	0.34318	0.05702	0.37544	0.82316	0.07379	0.81016	0.05215
52	11.1135	2.0994	12.0672	0.4731	0.26284	0.35539	0.04958	0.38820	0.80533	0.08298	0.80636	0.04473
53	10.7886	2.1684	11.8777	0.3969	0.27410	0.36787	0.04307	0.40116	0.78577	0.09318	0.80078	0.03827
54	10.4582	2.2373	11.6727	0.3321	0.28574	0.38063	0.03737	0.41432	0.76433	0.10444	0.79346	0.03264
55	10.1228	2.3057	11.4542	0.2772	0.29774	0.39366	0.03240	0.42766	0.74091	0.11682	0.78447	0.02774
56	9.7829	2.3734	11.2236	0.2309	0.31011	0.40694	0.02806	0.44117	0.71540	0.13035	0.77383	0.02350
57	9.4391	2.4400	10.9825	0.1918	0.32284	0.42046	0.02428	0.45484	0.68772	0.14506	0.76160	0.01983
58	9.0920	2.5052	10.7321	0.1589	0.33593	0.43421	0.02099	0.46866	0.65779	0.16093	0.74778	0.01666
59	8.7424	2.5685	10.4737	0.1313	0.34936	0.44819	0.01813	0.48260	0.62559	0.17790	0.73241	0.01392
60	8.3908	2.6295	10.2084	0.1082	0.36312	0.46236	0.01565	0.49667	0.59115	0.19589	0.71551	0.01158
61	8.0382	2.6878	9.9372	0.0890	0.37719	0.47671	0.01349	0.51083	0.55452	0.21472	0.69708	0.00957
62	7.6853	2.7430	9.6612	0.0729	0.39156	0.49124	0.01162	0.52508	0.51586	0.23419	0.67717	0.00786
63	7.3330	2.7945	9.3811	0.0596	0.40621	0.50590	0.01000	0.53939	0.47539	0.25397	0.65579	0.00641
64	6.9822	2.8421	9.0979	0.0486	0.42111	0.52070	0.00860	0.55373	0.43342	0.27370	0.63297	0.00518
65	6.6338	2.8851	8.8123	0.0395	0.43624	0.53559	0.00738	0.56810	0.39038	0.29288	0.60878	0.00415
66	6.2888	2.9231	8.5251	0.0320	0.45157	0.55056	0.00633	0.58247	0.34677	0.31096	0.58326	0.00329
67	5.9480	2.9558	8.2370	0.0259	0.46706	0.56559	0.00543	0.59681	0.30322	0.32730	0.55649	0.00257
68	5.6125	2.9827	7.9488	0.0208	0.48269	0.58064	0.00465	0.61111	0.26043	0.34119	0.52856	0.00199
69	5.2832	3.0034	7.6610	0.0167	0.49843	0.59568	0.00397	0.62534	0.21916	0.35193	0.49958	0.00152
70	4.9609	3.0177	7.3744	0.0134	0.51423	0.61070	0.00339	0.63947	0.18020	0.35881	0.46970	0.00114
71	4.6466	3.0252	7.0894	0.0107	0.53005	0.62566	0.00290	0.65349	0.14429	0.36123	0.43907	0.00084
72	4.3412	3.0257	6.8069	0.0085	0.54587	0.64052	0.00247	0.66736	0.11210	0.35871	0.40787	0.00061
73	4.0453	3.0190	6.5272	0.0067	0.56162	0.65527	0.00210	0.68107	0.08415	0.35101	0.37631	0.00043
74	3.7596	3.0051	6.2509	0.0053	0.57729	0.66987	0.00179	0.69459	0.06074	0.33813	0.34463	0.00030
75	3.4849	2.9838	5.9786	0.0042	0.59283	0.68430	0.00152	0.70791	0.04192	0.32040	0.31308	0.00020
76	3.2217	2.9552	5.7107	0.0033	0.60819	0.69851	0.00128	0.72099	0.02749	0.29844	0.28193	0.00013
77	2.9704	2.9195	5.4478	0.0025	0.62334	0.71249	0.00109	0.73382	0.01701	0.27312	0.25145	0.00009
78	2.7315	2.8769	5.1902	0.0020	0.63824	0.72621	0.00092	0.74638	0.00985	0.24552	0.22193	0.00005
79	2.5050	2.8275	4.9383	0.0015	0.65285	0.73964	0.00078	0.75865	0.00528	0.21679	0.19366	0.00003

Standard Service Table

x	l_x	w_x	i_x	r_x	d_x	x	l_x	w_x	i_x	r_x	d_x	
35	218,833.9	10,665.3	213.3	0	83.5	51	114,572.5	2,266.1	113.3	0	150.9	
36	207,871.8	10,130.9	202.6	0	83.6	52	112,042.2	2,215.9	110.8	0	162.8	
37	197,454.7	9,623.1	192.5	0	84.0	53	109,552.7	2,166.5	108.3	0	176.0	
38	187,555.1	9,140.6	182.8	0	84.7	54	107,101.9	2,117.8	105.9	0	190.5	
39	178,147.0	8,681.9	173.6	0	85.7	55	104,687.7	2,069.9	103.5	0	206.4	
40	169,205.8	8,246.0	164.9	0	86.9	56	102,307.9	2,022.6	101.1	0	223.9	
41	160,707.9	7,831.8	156.6	0	88.5	57	99,960.2	1,976.0	98.8	0	243.2	
42	152,631.0	7,438.0	148.8	0	90.5	58	97,642.2	1,929.9	96.5	0	264.4	
43	144,953.7	7,063.7	141.3	0	92.7	59	95,351.5	1,884.3	94.2	0	287.6	
44	137,656.1	6,707.9	134.2	0	95.3	60	93,085.4	0	0	27,925.6	0	Exact Age
45	130,718.7	2,586.1	129.3	0	99.7	60	65,159.8	0	61.9	6,187.6	210.4	
46	127,903.5	2,530.4	126.5	0	106.2	61	58,699.9	0	55.7	5,573.1	211.5	
47	125,140.4	2,475.6	123.8	0	113.4	62	52,859.6	0	50.2	5,017.5	212.7	
48	122,427.6	2,421.8	121.1	0	121.4	63	47,579.3	0	45.2	4,515.2	213.9	
49	119,763.2	2,369.0	118.5	0	130.3	64	42,805.0	0	40.6	4,061.0	215.1	
50	117,145.5	2,317.1	115.9	0	140.1	65	38,488.3	0	0	38,488.3	0	Exact Age

$w_x \rightarrow$ withdrawals; $i_x \rightarrow$ disability; $r_x \rightarrow$ retirements; $d_x \rightarrow$ deaths