MAP 4175 / 5177
Quiz 11

Name: KEY

Date: November 18, 2015

Show all work for full credit, and use correct notation. Simplify answers completely.

The non-zero transition rates for a 4-state model are:

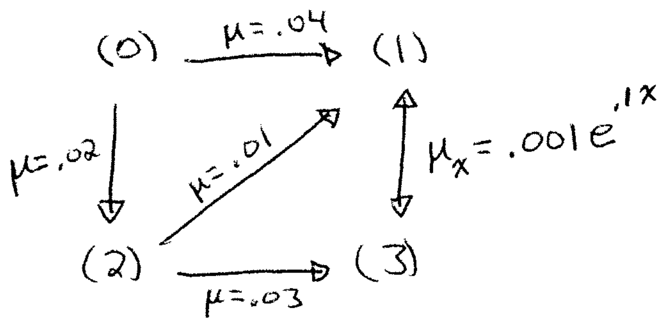
$\mu_x^{01} = .04$

$\mu_x^{02} = .02$

$\mu_x^{21} = .01$

$\mu_x^{23} = .03$

$\mu_x^{13} = .001e^{0.1x} = \mu_x^{31}$



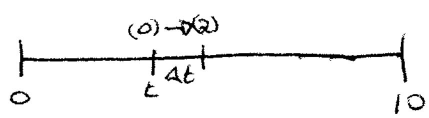

Determine

1. ${}_{10}p_{30}^{12}$ $= 0$

2. Show that ${}_{10}p_{30}^{00} = e^{-0.6} \approx 0.5488$

$${}_{10}P_{30}^{00} = {}_{10}P_{30}^{\overline{00}} = e^{-\int_0^{10}(.04+.02)\,dt} = e^{-.06(10)} = e^{-.6}$$

3. Show that ${}_{10}p_{30}^{02} = e^{-0.4} - e^{-0.6} \approx 0.1215$




$$Pr \doteq {}_tP_{30}^{00} \cdot \mu_{30+t}^{02} \cdot \Delta t \cdot {}_{10-t}P_{30+t}^{22} \text{ (Integrand)}$$

$${}_tP_{30}^{00} = {}_tP_{30}^{\overline{00}} = e^{-\int_0^t (.04+.02)\,dr} = e^{-.06t}$$

$${}_{10-t}P_{30+t}^{22} = {}_{10-t}P_{30+t}^{\overline{22}} = e^{-\int_0^{10-t}(.01+.03)\,dr} = e^{-.04(10-t)} = e^{-.4}\cdot e^{.04t}$$

$$\therefore\ {}_{10}P_{30}^{02} = \int_0^{10} e^{-.06t}(.02)\cdot e^{-.4}\cdot e^{.04t}\,dt = \int_0^{10} .02\cdot e^{-.4}\cdot e^{-.02t}\,dt$$

$$= e^{-.4}\cdot e^{-.02t}\Big|_{10}^{0} = e^{-.4}(1-e^{-.2}) = e^{-.4}-e^{-.6}$$

You are also given ${}_{10}p_{30}^{01} \approx 0.2587$ and ${}_{10}p_{30}^{03} \approx 0.0710$.

4. Determine ${}_{10}\dot{p}_{30}^{03}$ = "rate in" — "rate out"

$$ {}_t\dot{P}_{30}^{03} = \left( {}_tP_{30}^{02} \cdot \mu_{30+t}^{23} + {}_tP_{30}^{01} \cdot \mu_{30+t}^{13} \right) - \left( {}_tP_{30}^{03} \cdot \mu_{30+t}^{31} \right) $$

$$ \therefore \ {}_{10}\dot{P}_{30}^{03} = \underbrace{{}_{10}P_{30}^{02}}_{= \cancel{\#3} \ \doteq .1215} \cdot \underbrace{\mu_{40}^{23}}_{=.03} + \underbrace{{}_{10}P_{30}^{01}}_{=.2587} \cdot \underbrace{\mu_{40}^{13}}_{=.001(e^{4})} - \underbrace{{}_{10}P_{30}^{03}}_{=.0710} \cdot \underbrace{\mu_{40}^{31}}_{=.001e^{4}} $$

$$ \therefore \ {}_{10}\dot{P}_{30}^{03} \approx .1215(.03) + .2587(.001e^{4}) - .071(.001e^{4}) = .0139 $$

5. Use Euler's Forward Equation with step size equal to 0.2 to approximate ${}_{10.2}p_{30}^{03}$

$$ y(t+h) \approx y(t) + h \cdot \dot{y}(t) $$

$t = 10$

$h = .2$

$$ y(10.2) \approx y(10) + .2 \cdot \dot{y}(10) $$

$$ {}_{10.2}P_{30}^{03} = {}_{10}P_{30}^{03} + .2 \cdot {}_{10}\dot{P}_{30}^{03} $$

$$ = .071 + .2(.0139) = .07378 $$