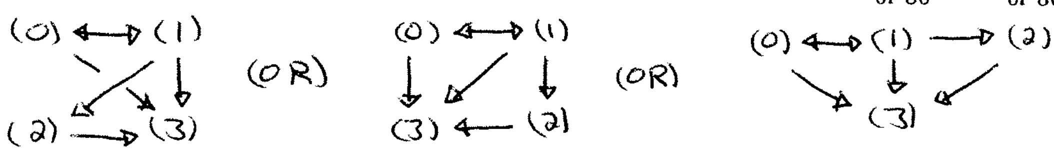
MAP 4175 / 5177 Name:________________________

 Date: November 30, 2016

Show all work for full credit, and use correct notation. Simplify answers completely.

A long-term care provider offers three care levels; Level 0 Care, Level 1 Care, and Level 2 Care. From Level 0 Care, a patient can only transfer to Level 1 Care. From Level 1 Care, a patient can transfer to either Level 0 Care or Level 2 Care. A patient in Level 2 Care will remain in Level 2 Care until death. Of course a patient can die while in any of the care levels. Define a 4-state model in which state (i) corresponds to a patient being in Level i Care, for $i = 0$, 1, and 2, and state (3) being the state that a person is dead.

1. Draw the schematic diagram for the model and state the values of ${}_0p_{80}^{10}$ and ${}_0p_{80}^{11}$.



$${}_0P_{80}^{10} = 0 \qquad {}_0P_{80}^{11} = 1$$

For Numbers 2-5 use the following transition intensities and probabilities:

t	${}_tp_{80}^{11}$	μ_{80+t}^{01}	μ_{80+t}^{03}	μ_{80+t}^{10}	μ_{80+t}^{12}	μ_{80+t}^{13}	μ_{80+t}^{23}
0	1.00000	0.10000	0.02981	0.08000	0.15000	0.05962	0.11924
1/3	0.90346	0.10000	0.03082	0.08000	0.15000	0.06164	0.12328
2/3	0.81652	0.10000	0.03186	0.08000	0.15000	0.06373	0.12746
1	--	0.10000	0.03294	0.08000	0.15000	0.06589	0.13178

2. Write down the Kolmogorov Differential Equation (KDE) for ${}_t\dot{p}_{80}^{10}$ and use it, along with the table values and the results from Problem 1, to show that ${}_0\dot{p}_{80}^{10} = 0.08$.

$${}_t\dot{P}_{80}^{10} = [\text{rate in}] - (\text{rate out})$$

$$\therefore \quad {}_t\dot{P}_{80}^{10} = {}_tP_{80}^{11} \cdot \mu_{80+t}^{10} - {}_tP_{80}^{10}\left(\mu_{80+t}^{01} + \mu_{80+t}^{03}\right)$$

$$\Longrightarrow \quad {}_0\dot{P}_{80}^{10} = {}_0P_{80}^{11}\,\mu_{80}^{10} - {}_0P_{80}^{10}\left(\mu_{80}^{01} + \mu_{80}^{03}\right)$$

$$= 1(.08) - 0(\text{———}) = .08$$

3. Using Euler's Method with a step-size of $h = 1/3$ and the results from the previous problems, show that ${}_{1/3}p_{80}^{10} = 0.02667$.

EM $(h = 1/3)$ $y(t + 1/3) = y(t) + \frac{1}{3} \cdot \dot{y}(t)$

$${}_{1/3}P_{80}^{10} = {}_{0}P_{80}^{10} + \frac{1}{3}\,{}_{0}\dot{P}_{80}^{10} = 0 + \frac{1}{3}(.08) = .02667$$

4. Using the KDE in Problem 2, along with the table values and the results from previous problems, show that ${}_{1/3}\dot{p}_{80}^{10} = 0.06879$. Then use a second iteration of Euler's Method with a step-size of $h = 1/3$ to show that ${}_{2/3}p_{80}^{10} = 0.04960$.

$${}_{1/3}\dot{P}_{80}^{10} \overset{KDE}{=} {}_{1/3}P_{80}^{11} \cdot \mu_{80+1/3}^{10} - {}_{1/3}P_{80}^{10}\left(\mu_{80+1/3}^{01} + \mu_{80+1/3}^{03}\right)$$

$$= (.90346)(.08) - .02667(.1 + .03082) = .06879$$

$${}_{2/3}P_{80}^{10} = {}_{1/3}P_{80}^{10} + \frac{1}{3}\left({}_{1/3}\dot{P}_{80}^{10}\right)$$

$$= .02667 + \frac{1}{3}(.06879) = .0496$$

5. Using the KDE in Problem 2, along with the table values and the results from previous problems, show that ${}_{2/3}\dot{p}_{80}^{10} = 0.05878$. Then use a third iteration of Euler's Method with a step-size of $h = 1/3$ to determine ${}_{1}p_{80}^{10}$.

$${}_{2/3}\dot{P}_{80}^{10} \overset{KDE}{=} {}_{2/3}P_{80}^{11}\, \mu_{80+2/3}^{10} - {}_{2/3}P_{80}^{10}\left(\mu_{80+2/3}^{01} + \mu_{80+2/3}^{03}\right)$$

$$= (.81652)(.08) - .0496(.1 + .03186) = .05878$$

$${}_{1}P_{80}^{10} = P_{80}^{10} = {}_{2/3}P_{80}^{10} + \frac{1}{3}\left({}_{2/3}\dot{P}_{80}^{10}\right)$$

$$= .0496 + \frac{1}{3}(.05878) = .06919$$