MAP 4175 / 5177 Name:________________________

Test 11 Date: November 14, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Given a two decrement model with $p_x'^{(1)} = 0.729$ and $p_x'^{(2)} = 0.81$, and assuming a constant force for each decrement during the year, determine $q_x^{(1)}$.

$p_x^{(\tau)} = (.729)(.81) = .59049$

$\Rightarrow q_x^{(\tau)} = .40951$

$p_x'^{(1)} \overset{CF}{=} \left[p_x^{(\tau)}\right]^{\left(q_x^{(1)}/q_x^{(\tau)}\right)}$

$\Rightarrow .729 = (.59049)^{q_x^{(1)}/.40951} \Rightarrow q_x^{(1)} = 0.245706$

(OR) $q_x^{(1)} = \int_0^1 {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(1)}\,dt$

$\overset{CF}{=} \mu^{(1)} \cdot \int_0^1 e^{-\mu^{(\tau)}t}\,dt = \frac{\mu^{(1)}}{\mu^{(\tau)}} \cdot e^{-\mu^{(\tau)}t}\Big|_1^0$

$= \frac{\mu^{(1)}}{\mu^{(\tau)}}\left[1 - e^{-\mu^{(\tau)}}\right] = \frac{\mu^{(1)}}{\mu^{(\tau)}} \cdot q_x^{(\tau)}$

$= \frac{-\ln(0.729)}{-\ln(0.59049)} \cdot (.40951)$

2. Given a two decrement model where each decrement is UDD in the double decrement table, if $q_x^{(1)} = 0.1$ and $q_x^{(2)} = 0.2$, determine ${}_{0.3}q_x'^{(2)}$

$q_x^{(\tau)} = 0.3 \Rightarrow p_x^{(\tau)} = 0.7$

${}_{.3}p_x'^{(2)} \overset{MUDD}{=} \left[{}_{.3}p_x^{(\tau)}\right]^{\left(q_x^{(2)}/q_x^{(\tau)}\right)}$ ${}_{.3}p_x^{(\tau)} = 1 - {}_{.3}q_x^{(\tau)} \overset{MUDD}{=} 1 - .3q_x^{(\tau)} = .91$

$\therefore {}_{.3}p_x'^{(2)} \overset{MUDD}{=} (.91)^{(2/3)}$

$\Rightarrow {}_{.3}q_x'^{(2)} = 1 - (.91)^{2/3} = 0.06093\ldots$

3. For a double decrement table where decrement 1 is SUDD and 30% of decrement 2 occurs at time 0.4 with the rest occurring at time 0.6, given $q_x'^{(1)} = 0.1$ and $q_x'^{(2)} = 0.2$ determine $q_x^{(1)}$ and $q_x^{(2)}$

$q_x^{(2)} = {}_{.4}p_x'^{(1)} \cdot (.3q_x'^{(2)}) + {}_{.6}p_x'^{(1)} \cdot (.7q_x'^{(2)})$

$= (1 - .4(.1)) \cdot (.06) + (1 - .6(.1)) \cdot (.14) = 0.1892$

$p_x^{(\tau)} = (.9)(.8) = .72 \Rightarrow q_x^{(\tau)} = .28 = q_x^{(1)} + 0.1892$

$q_x^{(1)} = .0908$

4. Given a two decrement model with $p_{60}'^{(1)} = p_{60}'^{(2)} = 0.80$ and $p_{61}'^{(1)} = p_{61}'^{(2)} = 0.75$, if each decrement is UDD in its associated single decrement model, determine ${}_{1.3}q_{60}^{(1)}$.

$${}_{1.3}q_{60}^{(1)} = q_{60}^{(1)} + p_{60}^{(\tau)} \cdot {}_{.3}q_{61}^{(1)} \qquad p_{60}^{(\tau)} = (.8)^2 = 0.64$$

$$q_{60}^{(1)} = \int_0^1 {}_tp_{60}'^{(2)} \cdot {}_tp_{60}'^{(1)} \cdot \mu_{60+t}^{(1)}\,dt \stackrel{\text{SUDD}}{=} q_{60}'^{(1)} \cdot \int_0^1 (1-.2t)\,dt$$

$$= 0.2\left[1 - \frac{.2}{2}\right] = 0.18$$

$${}_{.3}q_{61}^{(1)} = \int_0^{0.3} {}_tp_{61}'^{(2)} \cdot {}_tp_{61}'^{(1)} \cdot \mu_{61+t}^{(1)}\,dt \stackrel{\text{SUDD}}{=} q_{61}'^{(1)} \cdot \int_0^{0.3} (1-.25t)\,dt$$

$$= 0.25\left[0.3 - \frac{.25}{2}(0.3)^2\right] = 0.0721875$$

$$\therefore {}_{1.3}q_{60}^{(1)} = 0.18 + 0.64\,(0.0721875) = 0.2262$$

5. For a triple decrement table where decrement 2 and decrement 3 are each UDD in their associated single decrement tables, and decrement 1 is EOY, given $q_x'^{(j)} = 0.2j$ for $j = 1, 2$, and 3, determine $q_x^{(1)}$, $q_x^{(2)}$, and $q_x^{(3)}$

$$q_x'^{(1)} = .2 \qquad q_x'^{(2)} = .4 \qquad q_x'^{(3)} = .6$$

$$q_x^{(1)} \stackrel{\text{EOY}}{=} p_x'^{(2)} \cdot p_x'^{(3)} \cdot q_x'^{(1)} = (.6)(.4)(.2) = 0.048$$

$$q_x^{(2)} = \int_0^1 \underbrace{{}_tp_x'^{(1)}}_{\stackrel{\text{EOY}}{=} 1} \cdot \underbrace{{}_tp_x'^{(3)}}_{\stackrel{\text{SUDD}}{=} 1-.6t} \cdot \underbrace{{}_tp_x'^{(2)} \cdot \mu_{x+t}^{(2)}}_{\stackrel{\text{SUDD}}{=} q_x'^{(2)} = .4}\,dt = .4\int_0^1 (1-.6t)\,dt$$

$$\therefore q_x^{(2)} = .4\left(1 - \frac{.6}{2}\right) = 0.28$$

We can get $q_x^{(3)}$ just like we got $q_x^{(2)}$, or

$$q_x^{(3)} = q_x^{(\tau)} - q_x^{(1)} - q_x^{(2)} \qquad q_x^{(\tau)} = 1 - p_x^{(\tau)} = 1-(.8)(.6)(.4) = .808$$

$$\therefore q_x^{(3)} = 0.48$$