MAP 4175 / 5177 Name:______________________________

Test 11 Date: November 13, 2019

Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. You are given:
   (i) Decrement 1 is a beginning of year decrement and $q'^{(1)}_x = 0.1$
   (ii) Decrement 2 is an end of year decrement and $q'^{(2)}_x = 0.2$
   (iii) Decrement 3 is SUDD and $q'^{(3)}_x = 0.3$

   Determine $q^{(3)}_x$

   Solution:

   Since decrement 1 is BOY, $q^{(1)}_x = q'^{(1)}_x = 0.1$

   Since decrement 2 is EOY, $q^{(2)}_x = p'^{(1)}_x \cdot p'^{(3)}_x \cdot q'^{(2)}_x = 0.9 \cdot 0.7 \cdot 0.2 = 0.126$

   $p^{(\tau)}_x = p'^{(1)}_x \cdot p'^{(2)}_x \cdot p'^{(3)}_x = 0.9 \cdot 0.8 \cdot 0.7 = 0.504 \Rightarrow q^{(\tau)}_x = 0.496$

   $$\therefore q^{(3)}_x = q^{(\tau)}_x - q^{(1)}_x - q^{(2)}_x = 0.27$$

   There are several other correct ways to show that $q^{(3)}_x = 0.27$

2. Given $q^{(1)}_x = 0.1$ and $q^{(2)}_x = 0.2$, determine ${}_{0.25}\,p'^{(1)}_x$ using the MUDD assumption.

   Solution:

   The general MUDD formula is: ${}_tp'^{(j)}_x = \left({}_tp^{(\tau)}_x\right)^{\left(\frac{q^{(j)}_x}{q^{(\tau)}_x}\right)}$

   With $t = 0.25$ and $j = 1$, we have ${}_{0.25}p'^{(1)}_x = \left({}_{0.25}p^{(\tau)}_x\right)^{\left(\frac{q^{(1)}_x}{q^{(\tau)}_x}\right)}$

   $q^{(\tau)}_x = 0.1 + 0.2 = 0.3$ and ${}_{0.25}p^{(\tau)}_x = 1 - {}_{0.25}q^{(\tau)}_x$

   By the MUDD assumption, ${}_{0.25}q^{(\tau)}_x = 0.25 \cdot q^{(\tau)}_x = 0.25 \cdot 0.3 = 0.075$

   $$\therefore {}_{0.25}p'^{(1)}_x = (0.925)^{\left(\frac{0.1}{0.3}\right)} = \sqrt[3]{0.925} = 0.9743\cdots$$

For Numbers 3 – 5, you are given: $q_x'^{(1)} = 0.25$ and $q_x'^{(2)} = 0.20$

3. If there is a uniform distribution of departures in the associated single decrement models for decrements 1 and 2, then determine $q_x^{(1)}$

   Solution:
   Since SUDD and the implied duration value is 1, we can use the formula

$$q_x^{(1)} = q_x'^{(1)} \cdot \left(1 - \frac{q_x'^{(2)}}{2}\right) = 0.25 \cdot \left(1 - \frac{0.2}{2}\right) = 0.225$$

   Alternatively, the solution is similar to the solution to number 5 below.

4. If there is a uniform distribution of departures in the double decrement model, then determine $q_x^{(2)}$

   Solution:

   The general MUDD formula is: ${}_tp_x'^{(j)} = \left({}_tp_x^{(\tau)}\right)^{\left(\frac{q_x^{(j)}}{q_x^{(\tau)}}\right)}$

   With $t = 1$ and $j = 2$, we have $p_x'^{(2)} = \left(p_x^{(\tau)}\right)^{\left(\frac{q_x^{(2)}}{q_x^{(\tau)}}\right)}$ and after substitutions, we get

$$0.8 = (0.6)^{\left(\frac{q_x^{(2)}}{0.4}\right)} \Rightarrow q_x^{(2)} = 0.4 \cdot \frac{\ln(0.8)}{\ln(0.6)} = 0.17473 \cdots$$

5. Under the SUDD assumption for both decrements, determine ${}_{0.4}q_x^{(1)}$

   Solution:

$${}_{0.4}q_x^{(1)} = \int_0^{0.4} {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(1)} \cdot dt = \int_0^{0.4} {}_tp_x'^{(2)} \cdot {}_tp_x'^{(1)} \cdot \mu_{x+t}^{(1)} \cdot dt$$

   By SUDD, ${}_tp_x'^{(1)} \cdot \mu_{x+t}^{(1)} = \text{constant} = q_x'^{(1)} = 0.25$, and also by SUDD,

$${}_tp_x'^{(2)} = 1 - {}_tq_x'^{(2)} = 1 - t \cdot q_x'^{(2)} = 1 - 0.2 \cdot t$$

   So after substitution and factoring out the constant 0.25 from the integrand, we get

$${}_{0.4}q_x^{(1)} = 0.25 \cdot \int_0^{0.4} (1 - 0.2t) \cdot dt = 0.25 \cdot (t - 0.1t^2)|_0^{0.4} = 0.096$$