Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. You are given:
   (i) Decrement 1 is a beginning of year decrement and $q^{(1)}_x = 0.1$
   (ii) Decrement 2 is an end of year decrement and $q^{(2)}_x = 0.2$
   (iii) Decrement 3 is SUDD and $q^{(3)}_x = 0.3$

Determine $q^{(3)}_x$

Solution:
Since decrement 1 is BOY, $q^{(1)}_x = q^{(3)}_x = 0.1$

Since decrement 2 is EOY, $q^{(2)}_x = p^{(1)}_x \cdot p^{(2)}_x \cdot p^{(3)}_x = 0.9 \cdot 0.7 \cdot 0.2 = 0.126$

$p^{(r)}_x = p^{(1)}_x \cdot p^{(2)}_x \cdot p^{(3)}_x = 0.9 \cdot 0.8 \cdot 0.7 = 0.504 \Rightarrow q^{(r)}_x = 0.496$

$\therefore q^{(3)}_x = q^{(r)}_x - q^{(1)}_x - q^{(2)}_x = 0.27$

There are several other correct ways to show that $q^{(3)}_x = 0.27$

2. Given $q^{(1)}_x = 0.1$ and $q^{(2)}_x = 0.2$, determine $0.25p^{(1)}_x$ using the MUDD assumption.

Solution:

The general MUDD formula is: $tp^{(j)}_x = t \left( p^{(r)}_x \left( \frac{q^{(j)}_x}{q^{(1)}_x} \right) \right)$

With $t = 0.25$ and $j = 1$, we have $0.25p^{(1)}_x = 0.25 \left( \frac{q^{(j)}_x}{q^{(1)}_x} \right) = 0.25p^{(r)}_x = 0.25 \cdot 0.3 = 0.075$

$0.25p^{(1)}_x = (0.925) \left( \frac{0.1}{0.75} \right) = \frac{3}{0.925} = 0.9743 \cdots$
For Numbers 3 – 5, you are given: \( q_x^{(1)} = 0.25 \) and \( q_x^{(2)} = 0.20 \)

3. If there is a uniform distribution of departures in the associated single decrement models for decrements 1 and 2, then determine \( q_x^{(1)} \)

Solution:
Since SUDD and the implied duration value is 1, we can use the formula

$$ q_x^{(1)} = q_x^{(1)} \cdot \left(1 - \frac{q_x^{(2)}}{2}\right) = 0.25 \cdot \left(1 - \frac{0.2}{2}\right) = 0.225 $$

Alternatively, the solution is similar to the solution to number 5 below.

4. If there is a uniform distribution of departures in the double decrement model, then determine \( q_x^{(2)} \)

Solution:

The general MUDD formula is:

$$ tP_x^{(j)} = \left(\frac{q_x^{(j)}}{t q_x^{(j)}}\right) $$

With \( t = 1 \) and \( j = 2 \), we have:

$$ p_x^{(2)} = \left(\frac{q_x^{(2)}}{q_x^{(1)}}\right) $$

and after substitutions, we get

$$ 0.8 = (0.6) \cdot (0.6) \Rightarrow q_x^{(2)} = 0.4 \cdot \frac{\ln(0.8)}{\ln(0.6)} = 0.17473 \ldots $$

5. Under the SUDD assumption for both decrements, determine \( 0.4q_x^{(1)} \)

Solution:

$$ 0.4q_x^{(1)} = \int_0^{0.4} tP_x^{(1)} \cdot \mu_x \cdot dt = \int_0^{0.4} tP_x^{(2)} \cdot tP_x^{(1)} \cdot \mu_x \cdot dt $$

By SUDD, \( tP_x^{(1)} \cdot \mu_x = \text{constant} = q_x^{(1)} = 0.25 \), and also by SUDD,

$$ tP_x^{(2)} = 1 - tq_x^{(2)} = 1 - t \cdot q_x^{(2)} = 1 - 0.2 \cdot t $$

So after substitution and factoring out the constant 0.25 from the integrand, we get

$$ 0.4q_x^{(1)} = 0.25 \cdot \int_0^{0.4} (1 - 0.2t) \cdot dt = 0.25 \cdot \left( t - 0.1t^2 \right)|_0^{0.4} = 0.096 $$