MAP 4176 / 5178 Name:______________________

Test 12-13 Date: December 6, 2017

Show all work for full credit, use correct notation, and clearly mark your answer.

In each problem, use ILT actuarial assumptions to determine
(a) the expected present value of the insurance described
(b) the variance of the present value random variable of the insurance described
Each part of each problem is worth 10 points.

1. a 1-year discrete endowment insurance of 100,000 issued to (50)

$Z = 100000\, Z_{50:\overline{1}|} = 100000\, v$ (constant)

$\therefore$ EPV $= 100000\, v \overset{ILT}{=} \frac{100000}{1.06} = 94339.62\ldots$

$Var(Z) = 0$ since Z is constant

2. a 2-year term discrete insurance issued to (40), with a benefit of 1000 if death occurs in the first year, and 2000 if death occurs in the second year.

$v \overset{ILT}{=} \frac{1}{1.06}$

Z	Pr
$1000v$	$q_{40} \overset{ILT}{=} .00278$
$2000v^2$	${}_{1|}q_{40} = p_{40} \cdot q_{41} \overset{ILT}{=} (.99722)(.00298)$
0	${}_2p_{40} = p_{40} \cdot p_{41} \overset{ILT}{=} (.99722)(.99708)$

$EPV = E[Z] = 1000v\, q_{40} + 2000v^2 \cdot {}_{1|}q_{40} \overset{ILT}{=} 7.912\ldots$

$Var(Z) = E[Z^2] - (E[Z])^2$

$E[Z^2] = (1000v)^2 \cdot q_{40} + (2000v^2)^2 \cdot {}_{1|}q_{40} \overset{ILT}{=} 11889.698\ldots$

$\therefore Var(Z) = 11889.698\ldots - (7.912\ldots)^2 = 11827.094\ldots$

3. a 10-year deferred, 20-year term insurance issued to (25), with a benefit of 100 paid at the end of the year of death if (25) dies between ages 35 and 55.

$$EPV = E[Z] = 100 \cdot {}_{10}E_{25} \cdot A^{\,1}_{35:\overline{20|}} = 100 \cdot {}_{10}E_{25} \cdot (A_{35} - {}_{20}E_{35} \cdot A_{55})$$

$$\therefore\ EPV \overset{ILT}{=} 100 \cdot (.54997) \cdot (.12872 - .286(.30514))$$

$$\Rightarrow EPV = E[Z] \overset{ILT}{=} 2.279\ldots$$

$$E[Z^2] = 100^2 \cdot {}^{2}_{10}E_{25} \cdot ({}^{2}A_{35} - {}^{2}_{20}E_{35} \cdot {}^{2}A_{55})$$

$$\overset{ILT}{=} 100^2 \cdot (1.06)^{-10} \cdot {}_{10}E_{25} \cdot (.03488 - (1.06)^{-20} \cdot {}_{20}E_{35} \cdot (.13067))$$

$$\Rightarrow E[Z^2] \overset{ILT}{=} 71.331\ldots$$

$$\therefore\ Var(Z) = 71.331\ldots - (2.279\ldots)^2 = 66.134\ldots$$

4. a whole-life insurance issued to independent lives, both age 35, with benefit of 100 payable at the end of the year of the second death.

$$EPV = E[Z] = 100 A_{\overline{35:35}} = 100 (A_{35} + A_{35} - A_{35:35})$$

$$\therefore EPV \overset{ELT}{=} 100(.12872 \cdot (2) - .18694) = 7.05$$

$$E[Z^2] = 100^2 ({}^2A_{35} \cdot (2) - {}^2A_{35:35}) \overset{ILT}{=} 74.4$$

$$\therefore Var(Z) = 74.4 - (7.05)^2 = 24.6975$$

5. a 10-year term insurance issued to independent lives, ages 30 and 40, with benefit of 500 payable at the end of the year of the first death.

$$EPV = E[Z] = 500\left(A_{30:40} - {}_{10}E_{30:40} \cdot A_{40:50}\right)$$

$${}_{10}E_{30:40} = v^{10} \cdot {}_{10}p_{30:40} = v^{10} \cdot {}_{10}p_{30} \cdot {}_{10}p_{40}$$

$$\therefore\ EPV \overset{ILT}{=} 500\left(.19584 - v^{10}_{.06} \cdot \frac{l_{40}}{l_{30}} \cdot \frac{l_{50}}{l_{40}} \cdot (.29368)\right) = 20.675\ldots$$

$$E[Z^2] = 500^2\left({}^2A_{30:40} - {}_{10}^{2}E_{30:40} \cdot {}^2A_{40:50}\right)$$

$${}_{10}^{2}E_{30:40} = v^{20} \cdot {}_{10}p_{30} \cdot {}_{10}p_{40} = {}_{10}E_{30} \cdot {}_{10}E_{40}$$

$$\therefore\ E[Z^2] = 500^2\left({}^2A_{30:40} - {}_{10}E_{30} \cdot {}_{10}E_{40} \cdot {}^2A_{40:50}\right) \overset{ILT}{=} 7588.883\ldots$$

$$\therefore\ Var(Z) = 7588.883\ldots - (20.675\ldots)^2 = 7161.393\ldots$$