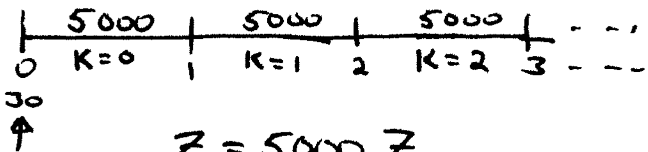
MAP 4176 / 5178 Name:______________________

Test 12 Date: December 7, 2016

Show all work for full credit, use correct notation., and clearly mark your answer. Use ILT actuarial assumptions (The ILT is posted on the projector.)

A whole life insurance of 5000 payable at the end of the year of death is issued to (30).

1. List out the first three values (the three largest values) of the present value random variable, Z, for this insurance, along with the associated probabilities for each value of Z.



$$v = \frac{1}{1.06}$$

$$Z = 5000\, Z_{30}$$

Z	Pr
$5000v = 4716.98$	$q_{30} = .00153$
$5000v^2 = 4449.98$	${}_{1\|}q_{30} = .00161$
$5000v^3 = 4198.10$	${}_{2\|}q_{30} = .00169$
$\vdots$	$\vdots$

$${}_{1|}q_{30} = p_{30} \cdot q_{31} = \frac{l_{31} - l_{32}}{l_{30}}$$

$${}_{2|}q_{30} = {}_2p_{30} \cdot q_{32} = \frac{l_{32} - l_{33}}{l_{30}}$$

2. Determine the single net premium for this insurance.

$$E[Z] = 5000\, E[Z_{30}] = 5000\, A_{30}$$

$$= 5000(.10248) = 512.40$$

3. Determine the variance of the present value random variable for this insurance.

$$Var(Z) = 5000^2 \cdot Var(Z_{30})$$

$$= 5000^2 \left[{}^2A_{30} - (A_{30})^2 \right]$$

$$= 5000^2 \left[.02531 - (.10248)^2 \right]$$

$$= 370196.24$$

4. Determine the probability that the present value random variable for this insurance is greater than 4750.

Z is at most $5000v = \frac{5000}{1.06} = 4716.98\cdots$

$\therefore \Pr(Z > 4750) = 0$

5. Determine the probability that the present value random variable for this insurance is less than 2500.

	K	Z	$\Pr$
$Z > 2500$	0	$5000v$	q_{30}
$Z > 2500$	1	$5000v^2$	${}_{1\|}q_{30}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$Z > 2500$	10	$5000v^{11}$	${}_{10\|}q_{30}$
$Z < 2500$	11	$5000v^{12}$	${}_{11\|}q_{30}$

$Z > 2500$

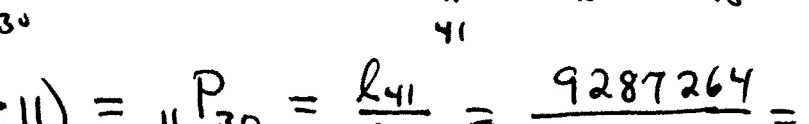
$$Z = 5000v^{K+1} < 2500$$

$$\Rightarrow v^{K+1} < \tfrac{1}{2}$$

$$\Rightarrow (1.06)^{K+1} > 2$$

$$\Rightarrow K > \frac{\ln(2)}{\ln(1.06)} - 1 = 10.8\cdots$$

$Z < 2500 \Rightarrow \Pr(Z < 2500) = \Pr(K \geq 11)$



$$\Pr(K \geq 11) = {}_{11}p_{30} = \frac{l_{41}}{l_{30}} = \frac{9287264}{9501381} = .97746\cdots$$

$\therefore \Pr(Z < 2500) = .97746\cdots$