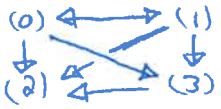
MAP 4175 / 5177 Name: _____

Quiz 12 – Take Home Date: November 28, 2018 – Due on November 29, 2018

Show all work for full credit, and use correct notation. Simplify answers completely.

A 4-state model has states: Healthy (0), Sick (1), Dead (2), and Terminally Ill (3).

The transition intensities are:



$\mu_x^{01} = 0.0001e^{.06x}$ $\mu_x^{02} = \mu_x^{12} = 6\mu_x^{01}$ $\mu_x^{03} = \mu_x^{13} = 0.05\mu_x^{01}$ $\mu_x^{10} = 0.1\mu_x^{01}$

$\mu_x^{32} = 1.2\mu_x^{02}$

1. Determine $1000 \cdot \mu_{40}^{1\tau}$

$\mu_{40}^{1\tau} = \mu_{40}^{10} + \mu_{40}^{12} + \mu_{40}^{13} = \mu_{40}^{01} \cdot (.1 + 6 + .05) = 6.15 \cdot \mu_{40}^{01}$

$\mu_{40}^{01} = .0001e^{.06(40)} = .0001e^{2.4}$

$\therefore 1000 \cdot \mu_{40}^{1\tau} = 1000 \cdot (6.15) \cdot (.0001e^{2.4}) = 0.615e^{2.4} = 6.779\ldots$

2. Determine $1000 \cdot C$, where C is such that $\mu_{40+t}^{32} = C \cdot e^{0.06t}$

$\mu_{40+t}^{32} = 1.2\,\mu_{40+t}^{02} = 1.2 \cdot (6\mu_{40+t}^{01}) = 7.2\,\mu_{40+t}^{01}$

$\mu_{40+t}^{01} = .0001e^{.06(40+t)} = .0001e^{.06(40)} \cdot e^{.06t} = .0001e^{2.4} \cdot e^{.06t}$

$\therefore \mu_{40+t}^{32} = 7.2 \cdot (.0001e^{2.4} \cdot e^{.06t}) = \underbrace{.00072e^{2.4}}_{=C} \cdot e^{.06t}$

$\therefore 1000 \cdot C = .72e^{2.4} = 7.936\ldots$

3. Determine ${}_5p_{40}^{30}$.

${}_5P_{40}^{30} = 0$ (There are no transitions from state 3 to state 0.)

4. Determine ${}_5p_{40}^{33}$

$${}_5P_{40}^{33} = {}_5P_{40}^{\overline{33}} = e^{-\int_0^5 \mu_{40+t}^{3\uparrow}\,dt} = e^{-\int_0^5 \mu_{40+t}^{32}\,dt}$$

$$\mu_{40+t}^{32} \overset{\text{see } \#2}{=\!=} c\cdot e^{.06t} \Rightarrow {}_5P_{40}^{33} = e^{-\int_0^5 c\cdot e^{.06t}\,dt}$$

$$\therefore {}_5P_{40}^{33} = e^{-\frac{c}{.06}e^{.06t}\Big|_0^5} = e^{-\frac{c}{.06}(e^{.3}-1)}$$

$$c \overset{\text{see } \#2}{=\!=} .00072e^{2.4}$$

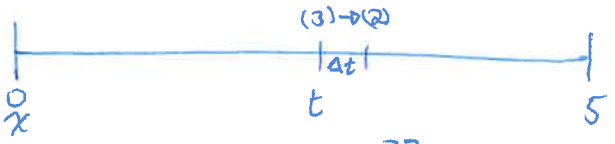
$$\therefore {}_5P_{40}^{33} = e^{-\frac{.00072e^{2.4}}{.06}\cdot(e^{.3}-1)} = 0.9547\ldots$$

5. Determine ${}_5p_{40}^{32}$

(Easy Way) ${}_5P_{40}^{32} = 1 - {}_5P_{40}^{30} - {}_5P_{40}^{31} - {}_5P_{40}^{33}$

$= 1 - 0 - 0 - 0.9547\ldots = 0.0452\ldots$

(Hard Way) ${}_5P_{40}^{32}$:



$$Pr = {}_tP_{40}^{33}\cdot\mu_{40+t}^{32}\cdot\Delta t$$

$$\therefore {}_5P_{40}^{32} = \int_0 {}_tP_{40}^{33}\cdot\mu_{40+t}^{32}\,dt$$

abracadabra $0.0452\ldots$