MAP 4175 / 5177 Name:______________________________

Test 12 Date: November 20, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

For Numbers 1 and 2, you are given a two decrement model with

$p_x^{\prime(1)} = 0.8$,

$p_x^{\prime(2)} = 0.6$, and

decrement 1 is uniformly distributed in its associated single decrement model.

1. Determine $q_x^{(1)}$ assuming decrement 2 is a discrete middle of the year decrement

   Note that $p_x^{(\tau)} = (0.8)(0.6) = 0.48$ and so $q_x^{(\tau)} = 0.52$

   Since decrement 2 is MOY and decrement 1 is SUDD, then

   $$q_x^{(2)} = {}_{0.5}p_x^{\prime(1)} \cdot q_x^{\prime(2)} = \left(1 - 0.5 \cdot q_x^{\prime(1)}\right) \cdot q_x^{\prime(2)} = 0.36$$

   $$\therefore q_x^{(1)} = 0.52 - 0.36 = 0.16$$

2. Determine $q_x^{(1)}$ assuming decrement 2 is a discrete decrement with 40% of the decrement occurring at time $t = 0.4$ and the rest occurring at time $t = 0.6$

   Since decrement 2 is discrete, we'll start with it. Using the same facts as in the solution to number 1 above, and based on the timing of decrement 2, we have

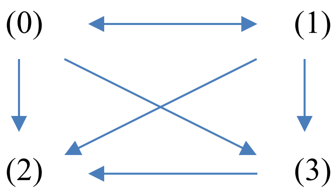
   $$q_x^{(2)} = {}_{0.4}p_x^{\prime(1)} \cdot \left(0.4 \cdot q_x^{\prime(2)}\right) + {}_{0.6}p_x^{\prime(1)} \cdot \left(0.6 \cdot q_x^{\prime(2)}\right)$$

   $$= \left(1 - 0.4 \cdot q_x^{\prime(1)}\right) \cdot \left(0.4 \cdot q_x^{\prime(2)}\right) + \left(1 - 0.6 \cdot q_x^{\prime(1)}\right) \cdot \left(0.6 \cdot q_x^{\prime(2)}\right) = 0.3584$$

   $$\therefore q_x^{(1)} = 0.52 - 0.3584 = 0.1616$$

For Numbers 3 through 5, you are given a four state model with

$\mu_x^{01} = 0.010 \quad \mu_x^{02} = \mu_x^{12} = 0.060 \quad \mu_x^{03} = \mu_x^{13} = 0.001 \quad \mu_x^{10} = 0.006 \quad \mu_x^{32} = 0.080$
All other forces of transition are equal to zero. (This is implied if not otherwise stated.)



3. Determine ${}_{10}p_x^{33}$

$$ {}_{10}p_x^{33} = {}_{10}p_x^{\overline{33}} = e^{-0.08(10)} = e^{-0.8} $$

4. Determine ${}_{10}p_x^{\overline{00}}$

$$ {}_{10}p_x^{\overline{00}} = e^{-(0.010+0.060+0.001)(10)} = e^{-0.71} $$

5. Determine ${}_{10}p_x^{32}$

$$ {}_{10}p_x^{32} = \int_0^{10} {}_tp_x^{\overline{33}} \cdot \mu_{x+t}^{32} dt = \int_0^{10} 0.08 \cdot e^{-0.08t} dt = e^{-0.08t}|_{10}^{0} = 1 - e^{-0.8} $$

Or, we could use the fact that ${}_{10}p_x^{32} + {}_{10}p_x^{33} = 1$ and from number 3, ${}_{10}p_x^{33} = e^{-0.8}$.