MAP 4175 / 5177
Test 13

Name: KEY

Date: December 5, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

Use the following schematic diagram for possible transitions between states for numbers 1 through 3.

$$\begin{tikzcd} (0) \arrow[r, leftrightarrow] \arrow[dr] & (1) \arrow[d, leftrightarrow] \\ (2) \arrow[r] & (3) \end{tikzcd}$$

$${}_t\dot{p}_x^{ij} = [\text{rate in}] - (\text{rate out})$$

1. Write an expression for the KDE ${}_t\dot{p}_x^{22}$

$${}_t\dot{p}_x^{22} = [0] - \left({}_tp_x^{22} \cdot \mu_{x+t}^{23}\right) = -{}_tp_x^{22} \cdot \mu_{x+t}^{23}$$

2. Write an expression for the KDE ${}_t\dot{p}_x^{23}$

$${}_t\dot{p}_x^{23} = \left[{}_tp_x^{20} \cdot \mu_{x+t}^{03} + {}_tp_x^{21} \cdot \mu_{x+t}^{13} + {}_tp_x^{22} \cdot \mu_{x+t}^{23}\right] - \left({}_tp_x^{23} \cdot \mu_{x+t}^{31}\right)$$

3. Write an expression for the KDE ${}_t\dot{p}_x^{10}$

$${}_t\dot{p}_x^{10} = \left[{}_tp_x^{11} \cdot \mu_{x+t}^{10}\right] - \left({}_tp_x^{10} \cdot \left(\mu_{x+t}^{01} + \mu_{x+t}^{03}\right)\right)$$

4. You are given:

$y(0) = 0$

$\dot{y}(0) = 0.09$

$\dot{y}(\frac{1}{3}) = 0.072 - 0.12 \cdot y(\frac{1}{3})$

$\dot{y}(\frac{2}{3}) = 0.0033 - 0.10 \cdot y(\frac{2}{3})$

Use Euler's Forward Method with step size $\frac{1}{3}$ to determine $y(1)$

$h = \frac{1}{3} \qquad y(t+\frac{1}{3}) = y(t) + \frac{1}{3} \cdot \dot{y}(t)$

$t=0: \quad y(1/3) = y(0) + \frac{1}{3} \cdot \dot{y}(0) = 0 + \frac{1}{3}(.09) = .03$

$t=1/3: \quad y(2/3) = y(1/3) + \frac{1}{3} \cdot \dot{y}(1/3) = .03 + \frac{1}{3}(.072 - .12(.03)) = .0528$

$t=2/3: \quad y(1) = y(2/3) + \frac{1}{3} \cdot \dot{y}(2/3) = .0528 + \frac{1}{3}(.0033 - .1(.0528)) = .05214$

$\therefore \; y(1) \approx 0.05214$

5. In a 3-state model, you are given:

$\mu_t^{01} = 0.5 + 0.6t$

$\mu_t^{10} = 0.2$

$\mu_t^{12} = 2^t$

$$\begin{tikzcd} (0) \arrow[r, leftrightarrow] & (1) \arrow[dl] \\ (2) & \end{tikzcd}$$

Use Euler's Forward Equation with step size 0.5 to approximate ${}_{0.5}p_0^{01}$

$h = 0.5 \qquad y(t+0.5) = y(t) + 0.5 \cdot \dot{y}(t) \qquad y(t) = {}_tP_0^{01}$

$t=0: \quad y(0) = {}_0P_0^{01} = 0$

$\dot{y}(t) = {}_t\dot{P}_0^{01} = [{}_tP_0^{00} \cdot \mu_t^{01}] - ({}_tP_0^{01} \cdot (\mu_t^{10} + \mu_t^{12}))$

$\Rightarrow \dot{y}(0) = [1 \cdot (.5)] - (0 \cdot (\text{—})) = .5$

$\therefore \; y(0.5) = {}_{.5}P_0^{01} = 0 + 0.5(.5) = 0.25$

${}_{0.5}P_0^{01} \approx 0.25$