

Show all work for full credit, and use correct notation.

1. Given $q_{80+k} = 0.1(k+1)$ for $k = 0$ and 1 , determine ${}_2q_{80}$.

$$\left. \begin{array}{l} q_{80} = .1 \\ q_{81} = .2 \end{array} \right\} \Rightarrow \begin{array}{l} P_{80} = .9 \\ P_{81} = .8 \end{array} \quad \therefore {}_2P_{80} = (.9)(.8) = .72$$

$$\Rightarrow {}_2q_{80} = .28$$

2. Given ${}_xP_0 = \left(\frac{100-x}{100}\right)^{0.5}$, $0 \leq x \leq 100$, determine ${}_{13}P_{51}$.

$${}_{64}P_0 = {}_{51}P_0 \cdot {}_{13}P_{51} \quad \begin{array}{l} {}_{64}P_0 = (.36)^{.5} = .6 \\ {}_{51}P_0 = (.49)^{.5} = .7 \end{array}$$

$$\therefore {}_{13}P_{51} = \frac{.6}{.7} = \frac{6}{7}$$

3. Given

x	q_x
70	0.3
71	0.4
72	0.5

determine the probability that a 70-year old dies between ages 71 and 72.

$$P = {}_{11}q_{70} = P_{70} \cdot q_{71} = (.7)(.4) = .28$$

4. Given ${}_kq_{96} = 0.1(k+1)$ for $k = 0, 1, 2$, and 3, determine $\text{Var}[\overbrace{\text{Min}(K_{96}, 1)}^{=Y}]$.

K_{96}	P_r
0	0.1
1	0.2
2	0.3
3	0.4

$$\Rightarrow$$

$\overbrace{\text{Min}(K_{96}, 1)}^{=Y}$	P_r	Y^2
0	0.1	0
1	0.9	1

$$\text{Var}(Y) = E[Y^2] - (E[Y])^2$$

$$E[Y^2] = 0.9 = E[Y]$$

$$\therefore \text{Var}(Y) = 0.9 - (0.9)^2 = 0.09$$

5. Given ${}_{10}p_{30} = 0.9$ and $\int_{10}^{20} f_{30}(t) dt = 0.1$, determine ${}_{20}p_{30}$.

$${}_{10|10}q_{30} = 0.1$$

$$0.1 = {}_{10|10}q_{30} = {}_{10}P_{30} - {}_{20}P_{30} = 0.9 - {}_{20}P_{30}$$

$$\Rightarrow {}_{20}P_{30} = 0.8$$