MAP 4175 / 5177 Name:______________________________

Quiz 1 Date: September 4, 2019

Show all work for full credit, and use correct notation.

1. Given ${}_{k|}q_{70} = 0.1(k+1)$ for $k = 0$ and 1, determine ${}_{2}p_{70}$

$${}_{2}q_{70} = q_{70} + {}_{1|}q_{70} = 0.1 + 0.2 = 0.3 \quad \Rightarrow \quad {}_{2}p_{70} = 0.7$$

2. Given $T_{40} = 52.82$, determine the value of the difference $K_{40}^{(4)} - K_{40}^{(12)}$

With $T_{40} = 52.82$, we have $K_{40}^{(4)} = K_{40}^{(12)} = 52.75$.

$$\therefore K_{40}^{(4)} - K_{40}^{(12)} = 0$$

3. Given $\int_0^{20} f_{70}(t)dt = 0.95$ and $\int_{30}^{\infty} f_{40}(t)dt = 0.2$, determine ${}_{30|20}q_{40}$

Note that $\int_0^{20} f_{70}(t)\,dt = {}_{20}q_{70}$ and $\int_{30}^{\infty} f_{40}(t)\,dt = {}_{30}p_{40}$.

Since ${}_{30|20}q_{40} = {}_{30}p_{40} \cdot {}_{20}q_{70}$, we get

$${}_{30|20}q_{40} = 0.95 \cdot 0.2 = 0.19$$

4. Given

x	q_x
90	0.3
91	0.4
92	0.5
93	0.6

determine the value of the deferred mortality probability ${}_{1|2}q_{91}$

$${}_{1|2}q_{91} = p_{91} - {}_{3}p_{91} = p_{91} - p_{91} \cdot p_{92} \cdot p_{93} = 0.6 - (0.6)(0.5)(0.4) = 0.48$$

5. Given $q_{80} = 0.1$ and $E[Min(K_{80}, 2)] = 1.62$, determine p_{81}

The probability table for the discrete random variable $Min(K_{80}, 2)$ is

K_{80}	$Min(K_{80}, 2)$	Pr
0	0	q_{80}
1	1	${}_{1|}q_{80} = p_{80} \cdot q_{81}$
≥ 2	2	${}_{2}p_{80} = p_{80} \cdot p_{81}$

Since $E[Min(K_{80}, 2)] = 1.62$ we have $1.62 = p_{80} \cdot q_{81} + 2 \cdot p_{80} \cdot p_{81}$

Since $q_{80} = 0.1$, then $p_{80} = 0.9$, and so $1.62 = 0.9 \cdot (1 - p_{81}) + 2 \cdot 0.9 \cdot p_{81}$

$$\therefore p_{81} = 0.8$$