MAP 4175 / 5177 Name: Key

Quiz 2 Date: September 9, 2015

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Given ${}_tp_x = \frac{100-x-t}{100-x}$, determine μ_{x+t}

$$\mu_{x+t} = \frac{-{}_t\dot{p}_x}{{}_tp_x} = \frac{-\left(\frac{-1}{100-x}\right)}{\frac{100-x-t}{100-x}} = \frac{1}{100-x-t}$$

2. Given ${}_tp_x = \left(\frac{100-x-t}{100-x}\right)^{3/2}$, determine μ_{x+t}

$$\mu_{x+t} = \frac{-{}_t\dot{p}_x}{{}_tp_x} = \frac{-\frac{3}{2}\cancel{\left(\frac{100-x-t}{100-x}\right)^{1/2}}\left(\frac{-1}{100-x}\right)}{\left(\frac{100-x-t}{100-x}\right)^{\cancel{3/2}\,1}} = \frac{3/2}{100-x-t} = \frac{3}{2(100-x-t)}$$

3. Given $\mu_{x+t} = \frac{3}{100-x-t}$ for $0 < t < 100-x$, determine ${}_tp_x$

$$ {}_tp_x = e^{-\int_0^t \frac{3}{100-x-r}dr} = e^{3\ln(100-x-r)\big|_0^t} = e^{3\ln(100-x-t)-3\ln(100-x)}$$

$$= e^{3\ln\left(\frac{100-x-t}{100-x}\right)} \implies {}_tp_x = \left(\frac{100-x-t}{100-x}\right)^3$$

4. Given $\mu_{30+t} = \frac{2}{70-t}$, $0 < t < 70$, determine ${}_{20}p_{60}$

$$ {}_{20}p_{60} = e^{-\int_{30}^{50}\frac{2}{70-t}dt} = e^{2\ln(70-t)\big|_{30}^{50}} = e^{2\ln(20)-2\ln(40)}$$

$$= e^{2\ln\left(\frac{20}{40}\right)} \implies {}_{20}p_{60} = \left(\frac{20}{40}\right)^2 = .25$$

5. Given $\underbrace{\int_5^{10} {}_tp_{20}\mu_{20+t}dt}_{= {}_{5|5}q_{20}} = \frac{1}{12}$ and ${}_5p_{25} = \frac{10}{11}$, determine ${}_5q_{20}$

$$ {}_{5|5}q_{20} = {}_5p_{20} \cdot {}_5q_{25} \qquad {}_5q_{25} = 1 - {}_5p_{25} = \frac{1}{11}$$

$$\therefore \quad \frac{1}{12} = {}_5p_{20} \cdot \frac{1}{11}$$

$$\implies {}_5p_{20} = \frac{11}{12} \implies {}_5q_{20} = \frac{1}{12}$$