MAP 4175 / 5177 Name:______________________

Quiz 2 Date: September 20, 2017

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. Given ${}_tp_x = (.95)^t$, determine μ_{x+t}

$${}_t\dot{p}_x = (.95)^t \cdot \ln(.95)$$

$$\therefore \quad \mu_{x+t} = \frac{-{}_t\dot{p}_x}{{}_tp_x} = \frac{-\cancel{(.95)^t}\cdot \ln(.95)}{\cancel{(.95)^t}} = -\ln(.95)$$

2. Given $\mu_x = \frac{1}{100-x}$, $0 < t < 100$, determine ${}_{20}p_{10}$

$${}_{20}p_{10} = e^{-\int_{10}^{30}\mu_x\,dx} = e^{-\int_{10}^{30}\frac{1}{100-x}\,dx} = e^{+\ln(100-x)\Big|_{10}^{30}}$$

$$= e^{\ln(70)-\ln(90)} = e^{\ln\left(\frac{7}{9}\right)} = \frac{7}{9}$$

3. Given $\int_{50}^{54}\mu_x\,dx = .1$ and $\int_0^5 {}_tp_{50}\mu_{50+t}dt = .1$, determine q_{54}

$$\underbrace{\int_{50}^{54}\mu_x\,dx = .1}_{\Rightarrow\ {}_4p_{50} = e^{-.1}} \qquad \underbrace{\int_0^5 {}_tp_{50}\mu_{50+t}dt = .1}_{\Rightarrow\ {}_5q_{50} = .1\ \Rightarrow\ {}_5p_{50} = .9}$$

$$\therefore \quad {}_5p_{50} = {}_4p_{50}\cdot p_{54} \Rightarrow .9 = e^{-.1}\cdot p_{54}$$

$$\Rightarrow p_{54} = .9e^{.1}$$

$$\Rightarrow q_{54} = 1 - .9e^{.1}$$

4. Given $\mu_x^{ns} = \mu_x^s - .02$ and $p_x^{ns} = .95$, determine p_x^s

$$p_x^s = e^{-\int_x^{x+1} \mu_y^s \, dy} = e^{-\int_x^{x+1} (\mu_y^{ns} + .02)\, dy}$$

$$= \underbrace{e^{-\int_x^{x+1} \mu_y^{ns} \, dy}} \cdot \underbrace{e^{-\int_x^{x+1} (.02)\, dy}}$$

$$= p_x^{ns} \cdot e^{-.02}$$

$$\therefore \quad p_x^s = .95 \cdot e^{-.02}$$

5. Given $\mu_x^m = 1.2\mu_x^f$ and ${}_kp_x^f = .75$, determine ${}_kp_x^m$

$${}_kp_x^m = e^{-\int_x^{x+k} \mu_y^m \, dy} = e^{-\int_x^{x+k} 1.2\mu_y^f \, dy}$$

$$= \left[e^{-\int_x^{x+k} \mu_y^f \, dy} \right]^{1.2}$$

$$= \left[{}_kp_x^f \right]^{1.2} = (.75)^{1.2}$$