MAP 4175 / 5177
Quiz 2

Name: KEY

Date: September 12, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. Given $\mu_x = 0.03$, for all x, determine ${}_{20}p_{10}$

$${}_{20}p_{10} = e^{-\int_0^{20} \mu_{10+t}\,dt} = e^{-\int_0^{20} .03\,dt} = e^{-.03(20)} = 0.5488\ldots$$

OR

$${}_{20}p_{10} = e^{-\int_{10}^{30} \mu_x\,dx} = e^{-\int_{10}^{30} .03\,dx} = e^{-.03(30-10)} = 0.5488\ldots$$

2. Given $\int_{20}^{40} \mu_x\,dx = 0.1$ and $\int_0^5 \mu_{40+t}\,dt = 0.05$, determine ${}_{25}q_{20}$

used for ${}_{20}p_{20}$ — used for ${}_5p_{40}$

$${}_{20}p_{20} = e^{-\int_{20}^{40} \mu_x\,dx} = e^{-0.1} \qquad {}_5p_{40} = e^{-\int_0^5 \mu_{40+t}\,dt} = e^{-0.05}$$

$$\therefore\ {}_{25}p_{20} = {}_{20}p_{20} \cdot {}_5p_{40} = e^{-0.1} \cdot e^{-0.05} = e^{-0.15}$$

$$\therefore\ {}_{25}q_{20} = 1 - {}_{25}p_{20} = 1 - e^{-0.15} = 0.1392\ldots$$

3. Given $\int_0^{20} {}_tp_{50}\mu_{50+t}\,dt = 0.15$ and ${}_{5|15}q_{50} = 0.10$, determine $\int_0^5 {}_tp_{50}\mu_{50+t}\,dt$

$= \int_0^{20} f_{50}(t)\,dt$ $= {}_{20}q_{50}$

$= \int_0^5 f_{50}(t)\,dt$ $= {}_5q_{50}$

$${}_{20}q_{50} = {}_5q_{50} + {}_{5|15}q_{50}$$

$$\Rightarrow\ 0.15 = {}_5q_{50} + 0.10 \Rightarrow {}_5q_{50} = 0.05$$

$$\therefore\ \int_0^5 {}_tp_{50}\,\mu_{50+t}\,dt = 0.05$$

4. Given ${}_tp_{40} = \frac{60-t}{60}$, for $0 \le t \le 60$, determine $\overset{o}{e}_{40}$.

$$\overset{o}{e}_{40} = \int_0^{60} {}_tP_{40}\, dt = \int_0^{60} \left(\frac{60-t}{60}\right) dt$$

$$= \frac{1}{2}\left(\frac{60-t}{60}\right)^2 (+60)\Big|_0^{60} = 30$$

5. Given $e_{50} = 24.5$ and $e_{51} = 24.0$, determine q_{50}

$$e_{60} = P_{50}(1 + e_{51})$$

$$\therefore\ 24.5 = P_{50}(25) \Rightarrow P_{50} = 0.98$$

$$\therefore\ q_{50} = 0.02$$