MAP 4175 / 5177 Name:_____

Test 2 Date: September 11, 2019

Show all work for full credit, and use correct notation.

1. Given $\int_{85}^{89} \mu_t dt = 0.3$ and $p_{89} = 0.9$, determine ${}_5p_{85}$.

$$\int_{85}^{89} \mu_t dt = 0.3 \quad \Rightarrow \quad {}_4p_{85} = e^{-\int_{85}^{89} \mu_t dt} = e^{-0.3}$$

$$\therefore {}_5p_{85} = {}_4p_{85} \cdot p_{89} = e^{-0.3} \cdot (0.9) = 0.667$$

2. Given $\int_0^{10} {}_tp_{30}\mu_{30+t} dt = 0.3$ and ${}_5p_{35} = 0.84$, determine $\int_0^5 \mu_{30+t} dt$.

$$\int_0^{10} {}_tp_{30}\mu_{30+t} dt = {}_{10}q_{30} = 0.3 \quad \Rightarrow \quad {}_{10}p_{30} = 0.7$$

$${}_{10}p_{30} = 0.7 \text{ and } {}_5p_{35} = 0.84 \quad \Rightarrow \quad {}_5p_{30} = \frac{{}_{10}p_{30}}{{}_5p_{35}} = \frac{0.7}{0.84} = 0.8\bar{3} = e^{-\int_0^5 \mu_{30+t} dt}$$

$$\therefore \int_0^5 \mu_{30+t} dt = -\ln(0.8\bar{3}) = 0.182$$

3. Given ${}_tp_x = e^{-0.02t}$, determine $\overset{o}{e}_x$.

$$\overset{o}{e}_x = \int_0^\infty {}_tp_x dt = \int_0^\infty e^{-0.02t} dt = \frac{1}{0.02} = 50$$

4. Given ${}_tq_{70} = 1 - (0.9)^t$, determine e_{70}.

$${}_kp_{70} = 1 - {}_kq_{70} = (0.9)^k$$

$$\therefore e_{70} = \sum_{k=1}^{\infty} {}_kp_{70} = p_{70} + {}_2p_{70} + {}_3p_{70} + \cdots = 0.9 + 0.9^2 + 0.9^3 + \cdots = \frac{0.9}{1 - 0.9} = 9$$

5. Given $\mu_x = A + B \cdot c^x$ where $A = 0.00022$, $B = 0.0000027$, and $c = 1.124$, determine ${}_{10}p_{20}$.

$${}_{10}p_{20} = e^{-\int_{20}^{30} \mu_x dx} = e^{-\int_{20}^{30} (A + B \cdot c^x) dx} = e^{-10A} \cdot e^{\frac{-B}{\ln(c)} \cdot (c^{30} - c^{20})} = 0.997273$$