

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Given ${}_tq_x = \frac{t}{100-x}$, determine ${}_tP_{50} = \frac{50-t}{50}$ ${}_tP_{80} = \frac{20-t}{20}$

(a) $e_{50} = P_{50} + {}_2P_{50} + \dots$

$$= \frac{49}{50} + \frac{48}{50} + \dots + \frac{1}{50} = \frac{1}{50} (49+48+\dots+1) = \frac{1}{50} \cdot \left(\frac{49+1}{2} \cdot 49 \right)$$

$\therefore e_{50} = 24.5$

(b) μ_{80} $\mu_{80+t} = \frac{-\dot{{}_tP_{80}}}{{}_tP_{80}} = \frac{-\left(\frac{-1}{20}\right)}{\frac{20-t}{20}} = \frac{1}{20-t}$

$\therefore \mu_{80} = \frac{1}{20}$ (use $t=0$)

2. Given ${}_tp_x = \left(\frac{90-x-t}{90-x}\right)^2$, determine ${}_tP_{40} = \left(\frac{50-t}{50}\right)^2$ ${}_tP_{50} = \left(\frac{40-t}{40}\right)^2$

(a) μ_{40} $\mu_{40+t} = \frac{-\dot{{}_tP_{40}}}{{}_tP_{40}} = \frac{-2\left(\frac{50-t}{50}\right)\left(\frac{-1}{50}\right)}{\left(\frac{50-t}{50}\right)^2} = \frac{2/50}{\frac{50-t}{50}} = \frac{2}{50-t}$

$\therefore \mu_{40} = \frac{2}{50}$

(b) $e_{50}^{\circ} = \int_0^{\infty} {}_tP_{50} dt = \int_0^{40} \left(\frac{40-t}{40}\right)^2 dt$

$$= \frac{1}{3} \left(\frac{40-t}{40}\right)^3 (+40) \Big|_0^{40} = \frac{40}{3} (1)^3 - \frac{40}{3} (0)^3 = \frac{40}{3}$$

3. Given $l_x = 1000(0.9)^x$, determine ${}_{20}q_{30}$

$$\begin{aligned} {}_{20}q_{30} &= 1 - {}_{20}P_{30} = 1 - \frac{l_{50}}{l_{30}} = 1 - \frac{1000(.9)^{50}}{1000(.9)^{30}} \\ &= 1 - (.9)^{20} \end{aligned}$$

4. Given $\mu_x = \begin{cases} .03 & \text{if } 30 < x < 60 \\ .06 & \text{if } x \geq 60 \end{cases}$, determine ${}_{40}P_{45} = {}_{15}P_{45} \cdot {}_{25}P_{60}$

$$\therefore {}_{40}P_{45} = e^{-.03(15)} \cdot e^{-.06(25)} = e^{-1.95}$$

5. Use ILT assumptions to determine (round answer to the tenths place)

$$\begin{aligned} \text{(a) } 1000 \cdot ({}_{10}q_{25}) &= 1000 \frac{l_{25} - l_{35}}{l_{25}} \\ &= 1000 \frac{9565017 - 9420657}{9565017} = 15.092\dots \end{aligned}$$

$$\begin{aligned} \text{(b) } 1000 \cdot ({}_{5|10}q_{20}) &= 1000 \frac{l_{25} - l_{35}}{l_{20}} \\ &= 1000 \frac{9565017 - 9420657}{9617802} = 15.009\dots \end{aligned}$$