MAP 4175 / 5177 Name: KEY

Quiz 3 Date: September 27, 2017

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. Smokers have a constant force of mortality of 0.1, and non-smokers have a constant force of mortality of 0.05. For a population of 30-year olds, 10% are smokers and 90% are non-smokers. Determine q_{50} for this population of 30-year olds

$${}_np_x^s = e^{-.1n} \qquad {}_np_x^{ns} = e^{-.05n}$$

$$q_{50} = ratio_{50}^{s} \cdot q_{50}^{s} + ratio_{50}^{ns} \cdot q_{50}^{ns}$$

$$ratio_{50}^{s} = \frac{.1 \cdot {}_{20}p_{30}^{s}}{.1 \cdot {}_{20}p_{30}^{s} + .9 \cdot {}_{20}p_{30}^{ns}} = \frac{.1 \cdot e^{-2}}{.1e^{-2} + .9e^{-1}} = 0.0392\ldots$$

$$ratio_{50}^{ns} = 1 - ratio_{50}^{s} = 0.9607\ldots$$

$$\therefore q_{50} = (.0392\ldots)(1-e^{-.1}) + (.9607\ldots)(1-e^{-.05}) = .05059\ldots$$

2. Each individual has a constant force of mortality, μ, where μ is drawn from the uniform distribution on the interval [0.1,0.2]. Determine the value of ${}_{10}p_x$.

Given μ, ${}_{10}p_x = e^{-10\mu}$

$$\therefore \quad {}_{10}p_x = E[e^{-10\mu}] = \int_{.1}^{.2} e^{-10\mu} \cdot \frac{1}{.2-.1}\, d\mu$$

$$= \int_{.1}^{.2} 10e^{-10\mu}\, d\mu = +e^{-10\mu}\Big|_{.2}^{.1} = e^{-1} - e^{-2}$$

For Numbers 3 and 4, use the Illustrative Life Table to determine

3. ${}_{10}q_{25}$.

$${}_{10}q_{25} = 1 - {}_{10}p_{25} = 1 - \frac{l_{35}}{l_{25}} = 1 - \frac{9420657}{9565017}$$

$$\therefore \; {}_{10}q_{25} = 0.01509\ldots$$

4. ${}_{10|5}q_{25}$.

$${}_{10|5}q_{25} = \frac{l_{35} - l_{40}}{l_{25}} = \frac{9420657 - 9313166}{9565017} = 0.0112\ldots$$

5. Suppose the force of mortality is constant over the 2-year period centered at age 50. Determine the value of the force of mortality that is consistent with the mortality from the Illustrative Life Table.

$${}_2p_{49} \overset{CF}{=} e^{-2\mu} \qquad {}_2p_{49} \overset{ILT}{=} \frac{l_{51}}{l_{49}} = \frac{8897913}{9000057} = .9886\ldots$$

$$\therefore \; e^{-2\mu} = 0.9886\ldots$$

$$\Rightarrow \mu = \frac{\ln(0.9886\ldots)}{-2} = 0.0057\ldots$$