MAP 4175 / 5177 Name: KEY
Quiz 3 Date: September 18, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. Males have a constant force of mortality of 0.05 and females have a constant force of mortality of 0.04. For a population of 40-year olds, 40% are male. Determine q_{50} for this population of 40-year olds.

$$q_{50} = 1 - p_{50} \overset{CF}{=} 1 - e^{-\mu} \Rightarrow q_{50}^m = 1 - e^{-.05} \ \& \ q_{50}^f = 1 - e^{-.04}$$

$$q_{50} = ratio_{50}^m \cdot q_{50}^m + ratio_{50}^f \cdot q_{50}^f$$

$$ratio_{50}^m = \frac{.4 \cdot {}_{10}p_{40}^m}{.4 \cdot {}_{10}p_{40}^m + .6 \cdot {}_{10}p_{40}^f} \qquad {}_np_x \overset{CF}{=} e^{-\mu \cdot n} \Rightarrow \begin{matrix} {}_{10}p_{40}^m = e^{-.05(10)} \\ {}_{10}p_{40}^f = e^{-.04(10)} \end{matrix}$$

$$\therefore ratio_{50}^m = \frac{.4e^{-0.5}}{.4e^{-0.5} + .6e^{-0.4}} = 0.376\ldots \quad ratio_{50}^f = 1 - ratio_{50}^m = 0.623\ldots$$

$$\therefore q_{50} = (0.376\ldots)\cdot(1-e^{-.05}) + (0.623\ldots)\cdot(1-e^{-.04}) = 0.0428\ldots$$

2. Each individual has a constant force of mortality, μ, where μ is drawn from the uniform distribution on the interval [0.01,0.05]. Determine the value of ${}_{10}p_x$

Given μ, ${}_{10}p_x = e^{-10\mu}$

$$\therefore {}_{10}p_x = E[e^{-10\mu}] = \int_{0.01}^{0.05} e^{-10\mu} \cdot \frac{1}{0.05-0.01} \, d\mu$$

$$= \int_{0.01}^{0.05} 25e^{-10\mu} \, d\mu = \frac{25}{10} e^{-10\mu} \Big|_{0.05}^{0.01}$$

$$= 2.5(e^{-0.1} - e^{-0.5}) = 0.745\ldots$$

For Numbers 3 and 4, use the L-TAM Illustrative Life Table to determine

3. $_{10}q_{30}$ $= 1 - \dfrac{l_{40}}{l_{30}} \underset{\text{ILT}}{\overset{\text{L-TAM}}{=}} 0.0039\ldots$

4. $_{5|10}q_{30}$ $= \dfrac{l_{35} - l_{45}}{l_{30}} \underset{\text{ILT}}{\overset{\text{L-TAM}}{=}} 0.0052\ldots$

5. Suppose the force of mortality is constant over the 2-year period centered at age 40. Determine the value of the force of mortality that is consistent with the mortality from the L-TAM Illustrative Life Table.

$${}_2p_{39} \overset{\text{CF}}{=} e^{-2\mu} \qquad {}_2p_{39} \underset{\cancel{\text{ILT}}}{\overset{\text{Always}\ \cancel{\text{L-TAM}}}{=}} \frac{l_{41}}{l_{39}} \underset{\text{ILT}}{\overset{\text{L-TAM}}{=}} 0.9989\ldots$$

$$\therefore\ e^{-2\mu} = 0.9989\ldots$$

$$\Rightarrow \mu = -\tfrac{1}{2}\ln(0.9989\ldots) = 0.00051\ldots$$