MAP 4175 / 5177 Name:_____
Quiz 3 Date: September 18, 2019

Each problem is worth 10 points. Show all work for full credit, and use correct notation.

1. Smokers have a constant force of mortality of 0.10 and non-smokers have a constant force of mortality of 0.05. For a population of 30-year olds, 10% are smokers. Determine q_{50} for this population of 30-year olds.

For a smoker, ${}_{20}p_{30} = e^{-0.10(20)} = e^{-2}$

For a non-smoker, ${}_{20}p_{30} = e^{-0.05(20)} = e^{-1}$

For this population of 30-year olds,
the proportion of smokers at age 50 is $\frac{0.1e^{-2}}{0.1e^{-2}+0.9e^{-1}} = 0.03927\cdots$
and the proportion of non-smokers at age 50 is $\frac{0.9e^{-1}}{0.1e^{-2}+0.9e^{-1}} = 0.96072\cdots$

For a smoker, $q_{50} = 1 - e^{-0.10} = 0.09516\cdots$
For a non-smoker, $q_{50} = 1 - e^{-0.05} = 0.04877\cdots$

$$\therefore q_{50} = (0.09516\cdots)(0.03927\cdots) + (0.04877\cdots)(0.96072\cdots) = 0.05059\cdots$$

2. Each individual has a constant force of mortality, μ, where μ is drawn from the uniform distribution on the interval [0.02,0.07]. Determine the value of ${}_5q_x$

$${}_5p_x = \Pr(T_x > 5) = E[\Pr(T_x > 5|\mu)] = E[e^{-5\mu}] = \int_{0.02}^{0.07} e^{-5\mu} \cdot \frac{1}{0.07 - 0.02} d\mu$$
$$= 4(e^{-0.10} - e^{-0.35}) = 0.80059\cdots$$

$$\therefore {}_5q_x = 1 - 0.80059\cdots = 0.19940\cdots$$

3. At all ages, males have a force of mortality that is 10% higher than females. If ${}_{20}q_x = 0.2$ for females, determine ${}_{20}q_x$ for males.

$${}_{20}p_x^m = e^{-\int_x^{x+20} \mu_r^m dr} = e^{-\int_x^{x+20} 1.1\mu_r^f dr} = \left(e^{-\int_x^{x+20} \mu_r^f dr}\right)^{1.1} = \left({}_{20}p_x^f\right)^{1.1} = 0.8^{1.1}$$

$$\therefore {}_{20}q_x^m = 1 - 0.8^{1.1} = 0.21765\cdots$$

For Numbers 4 and 5, use the L-TAM SULT to determine

4. ${}_{10}q_{20}$

$${}_{10}q_{20} = \frac{l_{20} - l_{30}}{l_{20}} = 0.002727$$

5. ${}_{5|10}q_{20}$

$${}_{5|10}q_{20} = \frac{l_{25} - l_{35}}{l_{20}} = 0.003144$$