MAP 4175 / 5177 Name: KEY
Quiz 4 Date: September 23, 2015

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Using ILT mortality, determine each of the following:

(a) ${}_{0.5}q_{70}$ using the UDD assumption

$${}_{.5}q_{70} = 1 - {}_{.5}p_{70} = 1 - \frac{\ell_{70.5}}{\ell_{70}}$$

$$\ell_{70.5} \overset{\text{UDD}}{\underset{\text{ILT}}{=}} (.5)(6616155) + (.5)(6396609)$$

$$\Rightarrow {}_{.5}q_{70} \doteq .01659$$

Note: ${}_{.5}q_{70} \overset{\text{UDD}}{=} .5 \cdot q_{70}$

(b) ${}_{1.8}p_{72.2}$ using the CF assumption

$${}_{1.8}p_{72.2} = \frac{\ell_{74}}{\ell_{72.2}} \qquad \ell_{72.2} \overset{\text{CF}}{\underset{\text{ILT}}{=}} (6164663)^{.8}(5920394)^{.2}$$

$$\Rightarrow {}_{1.8}p_{72.2} = .926253$$

2. Given $\ell_{90} = 1000$, and $p_{90} = .8$, and ${}_2p_{90} = .45$ determine each of the following:

$$\ell_{91} = 800 \qquad \ell_{92} = 450$$

(a) $\ell_{90.45}$ using the UDD assumption

$$\ell_{90.45} \overset{\text{UDD}}{=} (.55)(1000) + .45(800) = 910$$

(b) $\ell_{91.5}$ using the CF assumption

$$\ell_{91.5} \overset{\text{CF}}{=} (800)^{.5}(450)^{.5} = \sqrt{800(450)} = 600$$

3. Given $q_{70} = 0.0104$ and $q_{71} = 0.0117$ determine ${}_{0.7}q_{70.6}$ using the CF assumption.

Let $\ell_{70} = 1000$

Then $\ell_{71} = 1000 \cdot p_{70} = 989.6$

and $\ell_{72} = \ell_{71} \cdot p_{71} = 978.02168$

$$\ell_{70.6} \overset{CF}{=} (\ell_{70})^{.4}(\ell_{71})^{.6} \doteq 993.74696$$

$$\ell_{71.3} \overset{CF}{=} (\ell_{71})^{.7}(\ell_{72})^{.3} \doteq 986.112185$$

$${}_{.7}q_{70.6} = 1 - \frac{\ell_{71.3}}{\ell_{70.6}}$$

$$\therefore {}_{.7}q_{70.6} \doteq .00768$$

4. Given $q_{80+k} = .1 + .05k$, for $k = 0$ and 1, determine ${}_{1.3|0.7}q_{80}$ using the CF assumption.

$q_{80} = .1$

$q_{81} = .15$

Let $\ell_{80} = 1000$

Then $\ell_{81} = 1000 \cdot p_{80} = 900$

and $\ell_{82} = 900(.85) = 765$

$$\ell_{81.3} \overset{CF}{=} (\ell_{81})^{.7}(\ell_{82})^{.3} \doteq 857.17242$$

$${}_{1.3|0.7}q_{80} = \frac{\ell_{81.3} - \ell_{82}}{\ell_{80}}$$

$$\therefore {}_{1.3|0.7}q_{80} \doteq \frac{857.17242 - 765}{1000} \doteq .09217$$

5. Given ${}_{k|}q_{90} = .1(k + 1)$, for $k = 0$ and 1, determine ${}_{0.2|0.5}q_{90.8}$ using the UDD assumption

$q_{90} = .1$

${}_{1|}q_{90} = .2$

Let $\ell_{90} = 1000$

Then $\ell_{91} = 1000(.9) = 900$

$$\ell_{92} = \ell_{90} \cdot {}_2p_{90} = \ell_{90} \cdot {}_2q_{90}$$

$${}_2q_{90} = q_{90} + {}_{1|}q_{90} = .1 + .2 = .3$$

$$\therefore \ell_{92} = 1000(.7) = 700$$

$$\ell_{90.8} = 1000(.2) + 900(.8) = 920$$

$$\ell_{91.5} = 900(.5) + 700(.5) = 800$$

$${}_{0.2|0.5}q_{90.8} = \frac{\ell_{91} - \ell_{91.5}}{\ell_{90.8}}$$

$$\therefore {}_{0.2|0.5}q_{90.8} = \frac{900 - 800}{920}$$

$$= \frac{100}{920} \doteq .1087$$