MAP 4175 / 5177 Name:_____
Test 4 Date: October 4, 2017

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Using ILT mortality, determine each of the following:

(a) ${}_{0.5}q_{60}$ using the CF assumption

$$0.5\,q_{60} = 1 - {}_{0.5}p_{60} = 1 - \frac{l_{60.5}}{l_{60}}$$

$l_{60} = 8188074$

$l_{61} = 8075403$

$$\therefore\ l_{60.5} \overset{CF}{=} l_{60}^{.5} \cdot l_{61}^{.5} = \sqrt{(8188074)(8075403)}$$

$$\Rightarrow {}_{0.5}q_{60} = 0.0069\ldots$$

(b) ${}_{1.3}p_{42.2}$ using the UDD assumption

$${}_{1.3}p_{42.2} = \frac{l_{43.5}}{l_{42.2}}$$

$l_{42} = 9259571$

$l_{43} = 9229925$

$l_{44} = 9198149$

$$l_{43.5} \overset{UDD}{=} .5 \cdot l_{43} + .5 \cdot l_{44} = 9214037$$

$$l_{42.2} \overset{UDD}{=} .8 \cdot l_{42} + .2 \cdot l_{43} = 9253641.8$$

$$\therefore\ {}_{1.3}p_{42.2} = 0.9957\ldots$$

2. Given $l_{90} = 1000$, and $p_{90} = 0.8$, and ${}_2p_{90} = 0.56$ determine each of the following:

$l_{91} = 1000(.8) = 800$

$l_{92} = 1000(.56) = 560$

(a) $l_{90.25}$ using the CF assumption

$$l_{90.25} \overset{CF}{=} l_{90}^{.75} \cdot l_{91}^{.25} = (1000)^{.75} \cdot (800)^{.25} = 945.741\ldots$$

(b) $l_{91.75}$ using the UDD assumption

$$l_{91.75} \overset{UDD}{=} .25\, l_{91} + .75\, l_{92}$$

$$= .25(800) + .75(560) = 620$$

3. Given $q_{70} = 0.0104$ and $q_{71} = 0.0117$ determine ${}_{1.25}q_{70.5}$ using the CF assumption.

$p_{70} = 0.9896$ $p_{71} = 0.9883$

Suppose $\ell_{70} = 1000$

$\ell_{71} = 989.6$

$\ell_{72} = 978.02168$

$${}_{1.25}q_{70.5} = 1 - \frac{\ell_{71.75}}{\ell_{70.5}}$$

$$\ell_{70.5} \overset{CF}{=} \ell_{70}^{.5} \cdot \ell_{71}^{.5} = 994.786\ldots$$

$$\ell_{71.75} \overset{CF}{=} \ell_{71}^{.25} \cdot \ell_{72}^{.75} = 980.903\ldots$$

$$\therefore {}_{1.25}q_{70.5} = 0.0139\ldots$$

4. Given $q_{80+k} = .1 + .05k$, for $k = 0$ and 1, determine ${}_{0.7|1.3}q_{80}$ using the UDD assumption.

$q_{80} = .1$ $q_{81} = .15$

$p_{80} = .9$ $p_{81} = .85$

$${}_{0.7|1.3}q_{80} = \frac{\ell_{80.7} - \ell_{82}}{\ell_{80}}$$

Suppose $\ell_{80} = 1000$

$\ell_{81} = 900$

$\ell_{82} = 765$

$$\ell_{80.7} \overset{UDD}{=} .3 \cdot \ell_{80} + .7 \cdot \ell_{81} = 930$$

$$\therefore {}_{0.7|1.3}q_{80} = 0.165$$

5. Given ${}_{k|}q_{90} = .1(k + 1)$, for $k = 0$ and 1, determine ${}_{0.2|0.5}q_{90.8}$ using the UDD assumption

$q_{90} = .1$ ${}_{1|}q_{90} = .2$

$p_{90} = .9$ $.2 = .9 \cdot q_{91} \Rightarrow q_{91} = \frac{2}{9}$

$p_{91} = \frac{7}{9}$

$${}_{0.2|0.5}q_{90.8} = \frac{\ell_{91} - \ell_{91.5}}{\ell_{90.8}}$$

Suppose $\ell_{90} = 1000$

$\ell_{91} = 900$

$\ell_{92} = 700$

$$\ell_{90.8} \overset{UDD}{=} .2\ell_{90} + .8 \cdot \ell_{91} = 920$$

$$\ell_{91.5} \overset{UDD}{=} .5\ell_{91} + .5\ell_{92} = 800$$

$$\therefore {}_{0.2|0.5}q_{90.8} = \frac{100}{920} = 0.10869\ldots$$