MAP 4175 / 5177 Name: Key
Test 4 Date: September 26, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Using L-TAM ILT mortality, determine each of the following:

(a) ${}_{0.5}p_{40}$ using the CF assumption

$$ {}_{0.5}P_{40} = \frac{\ell_{40.5}}{\ell_{40}} \qquad \ell_{40.5} \overset{CF}{=} \ell_{40}^{.5} \cdot \ell_{41}^{.5} = \sqrt{\ell_{40} \cdot \ell_{41}} $$

$$ \therefore \ {}_{0.5}P_{40} \ \underset{ILT}{\overset{L\text{-}TAM}{=}} \ 0.99973\cdots $$

(b) ${}_{3.2}q_{36.3}$ using the UDD assumption

$$ {}_{3.2}q_{36.3} = \frac{\ell_{36.3} - \ell_{39.5}}{\ell_{36.3}} \qquad \ell_{36.3} \overset{UDD}{=} .7 \cdot \ell_{36} + .3 \cdot \ell_{37} $$

$$ \ell_{39.5} \overset{UDD}{=} .5 \cdot \ell_{39} + .5 \cdot \ell_{40} = \frac{\ell_{39} + \ell_{40}}{2} $$

$$ \therefore \ {}_{3.2}q_{36.3} \ \underset{ILT}{\overset{L\text{-}TAM}{=}} \ 0.00143\cdots $$

2. Given $q_{70} = 0.010$ and $q_{71} = 0.012$ determine ${}_{1.25}q_{70.5}$ using the UDD assumption.

$p_{70} = 0.99 \qquad p_{71} = 0.988$

Assume $\ell_{70} = 1000$

$$ \ell_{71} = \ell_{70} \cdot p_{70} = 990 $$

$$ \therefore \ \ell_{72} = \ell_{71} \cdot p_{71} = 978.12 $$

$$ {}_{1.25}q_{70.5} = \frac{\ell_{70.5} - \ell_{71.75}}{\ell_{70.5}} $$

$$ \ell_{70.5} \overset{UDD}{=} .5 \cdot \ell_{70} + .5 \cdot \ell_{71} = \frac{\ell_{70} + \ell_{71}}{2} $$

$$ \ell_{71.75} \overset{UDD}{=} .25 \cdot \ell_{71} + .75 \cdot \ell_{72} $$

$$ \therefore \ {}_{1.25}q_{70.5} = 0.01397\cdots $$

3. Given $q_{80+k} = .1 + .05k$, for $k = 0$ and 1, determine ${}_{1.7|0.3}q_{80}$ using the CF assumption.

$q_{80} = 0.10 \qquad p_{80} = 0.90$

$q_{81} = 0.15 \qquad p_{81} = 0.85$

Assume $\ell_{80} = 1000$

$\ell_{81} = \ell_{80} \cdot p_{80} = 900$

$\ell_{82} = \ell_{81} \cdot p_{81} = 765$

$${}_{1.7|0.3}q_{80} = \frac{\ell_{81.7} - \ell_{82}}{\ell_{80}}$$

$$\ell_{81.7} \overset{CF}{=} \ell_{81}^{.3} \cdot \ell_{82}^{.7}$$

$\therefore \ {}_{1.7|0.3}q_{80} = 0.03822\ldots$

4. Given ${}_{k|}q_{90} = .1(k+1)$, for $k = 0$ and 1, determine ${}_{0.5|0.3}q_{90.6}$ using the UDD assumption

$q_{90} = 0.1 \qquad p_{90} = 0.9$

${}_{1|}q_{90} = p_{90} \cdot q_{91}$

$\Rightarrow 0.2 = 0.9 \cdot q_{91}$

$\therefore q_{91} = \frac{2}{9} \qquad p_{91} = \frac{7}{9}$

Assume $\ell_{90} = 1000$

$\ell_{91} = \ell_{90} \cdot p_{90} = 900$

$\ell_{92} = \ell_{91} \cdot p_{91} = 700$

$${}_{0.5|0.3}q_{90.6} = \frac{\ell_{91.1} - \ell_{91.4}}{\ell_{90.6}}$$

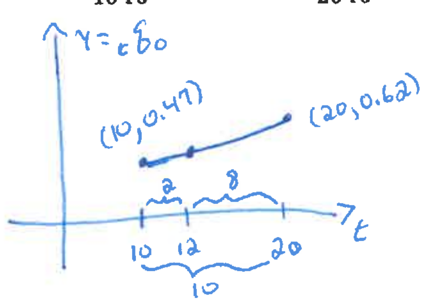
$$\ell_{90.6} \overset{UDD}{=} .4 \cdot \ell_{90} + .6 \cdot \ell_{91}$$

$$\ell_{91.1} \overset{UDD}{=} .9\,\ell_{91} + .1 \cdot \ell_{92}$$

$$\ell_{91.4} \overset{UDD}{=} .6 \cdot \ell_{91} + .4\,\ell_{92}$$

$\therefore \ {}_{0.5|0.3}q_{90.6} = 0.06382\ldots$

5. Given ${}_{10}q_0 = 0.47$ and ${}_{20}q_0 = 0.62$, use linear interpolation to determine ${}_{12}q_0$



weights are $\frac{2}{10}$ & $\frac{8}{10}$

$\therefore \ {}_{12}q_0 = \frac{8}{10} \cdot ({}_{10}q_0) + \frac{2}{10} \cdot ({}_{20}q_0)$

$\therefore \ {}_{12}q_0 = 0.5$