MAP 4175 / 5177 Name:________________________________

Quiz 5 Date: September 30, 2015

Show all work for full credit, and use correct notation. Simplify answers completely.

1. (15 points) For a 2-year select period, you are given:

$$q_{[20]} = 0.1$$

$$q_{[20]+1} = 0.2$$

$$d_{[x]} = \tfrac{1}{2} d_x \text{ for all } x$$

$$d_{[x]+1} = \tfrac{3}{4} d_{x+1} \text{ for all } x$$

Determine ${}_{1|}q_{20}$ $= \dfrac{\ell_{21} - \ell_{22}}{\ell_{20}}$

Let $\ell_{[20]} = 1000$

$\Rightarrow \ell_{[20]+1} = \ell_{[20]} \cdot p_{[20]} = 1000(.9) = 900$

$\Rightarrow \ell_{[20]+2} = \ell_{22} = \ell_{[20]+1} \cdot p_{[20]+1} = 900(.8) = 720$

$\ell_{[20]} = 1000$, $\ell_{[20]+1} = 900$ $\Rightarrow d_{[20]} = 100 \Rightarrow d_{20} = 200 = \ell_{20} - \ell_{21}$

$\ell_{[20]+1} = 900$, $\ell_{22} = \ell_{[20]+2} = 720$ $\Rightarrow d_{[20]+1} = 180 \Rightarrow d_{21} = \frac{4}{3}(180) = 240 = \ell_{21} - \ell_{22}$

$\therefore \ell_{21} = d_{21} + \ell_{22} = 240 + 720 = 960$

$\ell_{20} = d_{20} + \ell_{21} = 200 + 960 = 1160$

$$\therefore {}_{1|}q_{20} = \frac{960 - 720}{1160} = \frac{240}{1160}$$

2. (20 points) For a mortality table with a select period of two years, you are given:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
50	0.0050	0.0063	0.0080	52
51	0.0060	0.0073	0.0090	53
52	0.0070	0.0083	0.0100	54
53	0.0080	0.0093	0.0110	55

The force of mortality is constant between integral ages. Calculate $1000\,{}_{2.5}q_{[50]+0.4}$

$$ {}_{2.5}q_{[50]+.4} = \frac{l_{[50]+.4} - l_{52.9}}{l_{[50]+.4}} $$

Let $l_{[50]} = 1000$

$$ \Rightarrow l_{[50]+1} = l_{[50]} \cdot p_{[50]} = 1000(.995) = 995 $$

$$ \Rightarrow l_{[50]+2} = l_{52} = l_{[50]+1}\, p_{[50]+1} = 995(.9937) = 988.7315 $$

$$ \Rightarrow l_{53} = l_{52} \cdot p_{52} = 988.7315\,(.992) = 980.821648 $$

$$ l_{[50]+.4} \stackrel{CF}{=} \left(l_{[50]}\right)^{.6}\left(l_{[50]+1}\right)^{.4} = (1000)^{.6}(995)^{.4} = \boxed{1} $$

$$ l_{52.9} \stackrel{CF}{=} \left(l_{52}\right)^{.1}\left(l_{53}\right)^{.9} = \boxed{2} $$

$$ \therefore 1000\,{}_{2.5}q_{[50]+.4} = \frac{\boxed{1} - \boxed{2}}{\boxed{1}} \doteq 16.42 $$

3. (15 points) An ultimate mortality table follows $DML(\omega = 31)$. Find the probability that a person, insured one year ago at age 20, will die between ages 23 and 24, given

$$\left.\begin{array}{ll} q_{[x]+t} = \dfrac{t+1}{t+2} q_{x+t}, & t = 0, 1, 2 \\ q_{[x]+t} = q_{x+t}, & t = 3, 4, 5, \ldots \end{array}\right\} \text{3-year select period}$$

$${}_{2|}q_{[20]+1} = \frac{l_{23} - l_{24}}{l_{[20]+1}}$$

Let $l_{[20]+1} = 1000$

$$\Rightarrow l_{[20]+2} = l_{[20]+1} \cdot p_{[20]+1} = 1000\left(1 - \tfrac{2}{3} q_{21}\right)$$

$$q_{21} = \frac{1}{31-21} = \frac{1}{10}$$

$$\therefore l_{[20]+2} = 1000\left(\tfrac{14}{15}\right)$$

$$\Rightarrow l_{23} = l_{[20]+3} = l_{[20]+2} \cdot p_{[20]+2} = 1000\left(\tfrac{14}{15}\right)\left(1 - \tfrac{3}{4} q_{22}\right)$$

$$q_{22} = \frac{1}{31-22} = \frac{1}{9}$$

$$\therefore l_{23} = 1000\left(\tfrac{14}{15}\right)\left(\tfrac{11}{12}\right)$$

$$\Rightarrow l_{24} = l_{23} \cdot p_{23} = 1000\left(\tfrac{14}{15}\right)\left(\tfrac{11}{12}\right) \cdot \frac{31-23-1}{31-23} = 1000\left(\tfrac{14}{15}\right)\left(\tfrac{11}{12}\right)\left(\tfrac{7}{8}\right)$$

$$\therefore {}_{2|}q_{[20]+1} = \frac{\cancel{1000}\left(\frac{14}{15}\right)\left(\frac{11}{12}\right) - \cancel{1000}\left(\frac{14}{15}\right)\left(\frac{11}{12}\right)\left(\frac{7}{8}\right)}{\cancel{1000}} = \left(\tfrac{14}{15}\right)\left(\tfrac{11}{12}\right)\left(\tfrac{1}{8}\right)$$