

Show all work for full credit, and use correct notation. Simplify answers completely.

1. (15 points) For a 2-year select period, you are given:

$$q_{40} = q_{41} = q_{42} = 0.2$$

$$d_{[x]} = \frac{1}{2}d_x \text{ for all } x$$

$$d_{[x]+1} = \frac{3}{4}d_{x+1} \text{ for all } x$$

$$\text{Determine } {}_{1|2}q_{[40]} = \frac{l_{[40]+1} - l_{43}}{l_{[40]}}$$

$$\text{Let } l_{40} = 1000$$

$$\implies d_{40} = 200 \quad (\text{and } d_{[40]} = 100)$$

$$\implies l_{41} = 800$$

$$\implies d_{41} = 160 = d_{40+1} \quad (\text{and } d_{[40]+1} = 120)$$

$$\implies l_{42} = 640$$

$$\implies d_{42} = 128$$

$$\implies l_{43} = 512$$

$$\text{Also, } l_{42} = l_{[40]+1} - d_{[40]+1} \implies l_{[40]+1} = 760$$

$$\text{and } l_{[40]+1} = l_{[40]} - d_{[40]} \implies l_{[40]} = 860$$

$$\therefore {}_{1|2}q_{[40]} = \frac{760 - 512}{860} = \frac{248}{860} = 0.288\ldots$$

2. (20 points) For a mortality table with a select period of two years, you are given:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
50	0.050	0.065	0.080	52
51	0.055	0.070	0.085	53
52	0.060	0.075	0.090	54
53	0.065	0.080	0.095	55

There is a uniform distribution of deaths between integral ages.

$$\text{Calculate } 1000 {}_{1.5|}q_{[51]+0.5} = 1000 \cdot \frac{l_{53} - l_{54}}{l_{[51]+0.5}}$$

$$\text{Let } l_{[51]} = 1000 > l_{[51]+0.5} = \frac{1945}{2} = 972.5$$

$$l_{[51]+1} = 945$$

$$l_{53} = 945(1 - .07) = 878.85$$

$$l_{54} = 878.85(1 - .085) = 804.14775$$

$$\therefore 1000 {}_{1.5|}q_{[51]+0.5} = 1000 \frac{878.85 - 804.14775}{972.5} = 76.81 \dots$$

3. (15 points) An ultimate mortality table follows $DML(\omega = 40)$. Find the probability that a person, insured two years ago at age 32, will die between ages 35 and 36, given

$$q_{[x]+t} = \frac{t+1}{t+2} q_{x+t}, \quad t = 0, 1, 2$$

$$q_{[x]+t} = q_{x+t}, \quad t = 3, 4, 5, \dots$$

$${}_{11}q_{[32]+2} = P_{[32]+2} \cdot q_{35} \quad q_{34} = \frac{1}{6}$$

$$= \left(1 - \frac{3}{4} q_{34}\right) \cdot q_{35} \quad q_{35} = \frac{1}{5}$$

$$\therefore {}_{11}q_{[32]+2} = \left(1 - \frac{3}{4} \cdot \frac{1}{6}\right) \cdot \frac{1}{5} = \frac{7}{40}$$