MAP 4175 / 5177
Quiz 5

Name: KEY

Date: October 11, 2017

Show all work for full credit, and use correct notation. Simplify answers completely.
Each question is worth 10 points

1. For a mortality table with a select period of two years, you are given:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
50	0.050	0.065	0.080	52
51	0.055	0.070	0.085	53
52	0.060	0.075	0.090	54
53	0.065	0.080	0.095	55

Assume a constant force between integral ages.
Calculate $1000_{1.5|}q_{[51]+0.5}$ $= 1000 \cdot \dfrac{\ell_{53} - \ell_{54}}{\ell_{[51]+0.5}}$

Let $\ell_{[51]} = 1000$, $\ell_{[51]+1} = 945$ $\Rightarrow \ell_{[51]+0.5} \overset{CF}{=} \ell_{[51]}^{0.5} \cdot \ell_{[51]+1}^{0.5} = 972.11\ldots$

$\ell_{53} = 945(1-.07) = 878.85$

$\ell_{54} = 878.85(1-.085) = 804.14\ldots$

$\therefore 1000 \cdot {}_{1.5|}q_{[51]+0.5} = 1000 \cdot \dfrac{878.85 - 804.14\ldots}{972.11\ldots} = 76.84\ldots$

2. Given $q_x = 0.2$, and $e_{x:\overline{2}|} = 1.4$, determine q_{x+1}.

$p_x = 0.8$

$e_{x:\overline{2}|} = p_x + {}_2p_x = p_x \cdot (1 + p_{x+1})$

$\therefore 1.4 = 0.8(1+p_{x+1}) \Rightarrow p_{x+1} = 0.75$

$\therefore q_{x+1} = 0.25$

3. Suppose ${}_tp_{20:30} = \left(\frac{60-t}{60}\right)^2$ for $0 \le t \le 60$. $\mathring{e}_{20:30}$

Observe that mortality for (20:30) is GDML ($\alpha = 2$, "$\omega - x$" $= 60$)

$$\therefore \mathring{e}_{20:30} = \frac{\text{"}\omega - x\text{"}}{\alpha + 1} = \frac{60}{3} = 20$$

(OR) Using integrals,

$$\mathring{e}_{20:30} = \int_0^{60} {}_tp_{20:30}\, dt = \int_0^{60} \left(\frac{60-t}{60}\right)^2 dt$$

$$= \frac{1}{3}(-60)\left(\frac{60-t}{60}\right)^3 \Big|_0^{60} = 20\left(\frac{60-t}{60}\right)\Big|_{60}^{0} = 20$$

4. Given $\mu_x = 0.05$, determine $e_{x:\overline{30}|}$. ${}_tp_x \overset{CF}{=} e^{-.05n}$

$$e_{x:\overline{30}|} = p_x + {}_2p_x + \cdots + {}_{30}p_x$$

$$= e^{-.05} + e^{-.05(2)} + \cdots + e^{-.05(30)}$$

$$= v + v^2 + \cdots + v^{30} \quad \text{where } v = e^{-.05}$$

$$= a_{\overline{30}|i} \quad \text{where } i = \frac{1}{v} - 1 = e^{.05} - 1$$

$$\therefore e_{x:\overline{30}|} = 15.15\ldots$$

5. Given $e_{30:\overline{30}|} = 24$, $e_{30:\overline{17}|} = 15$, and $e_{47:\overline{13}|} = 10$, determine ~~$q_{30}$~~ ${}_{17}q_{30}$.

$$e_{30:\overline{30}|} = e_{30:\overline{17}|} + {}_{17}p_{30} \cdot e_{47:\overline{13}|}$$

$$\therefore \quad 24 = 15 + {}_{17}p_{30} \cdot 10$$

$$\Rightarrow {}_{17}p_{30} = \frac{24-15}{10} = 0.9$$

$$\therefore {}_{17}q_{30} = 0.1$$