MAP 4175 / 5177 Name: Key

Quiz 5 Date: October 3, 2018

Show all work for full credit, and use correct notation. Simplify answers completely.
Each question is worth 10 points

Numbers 1 and 2:
Given the following complete individual data for a mortality study on 10 dragons:

x_i: 3, 3, 3, 4, 5, 7, 7, 8, 9, 9

1. Use the empirical distribution to approximate the variance of the estimator used to approximate the probability that a dragon dies within 6 years.

$$\hat{F}_{10}(6) = \frac{N}{10} \qquad N \sim Bin(10, q)$$

$$Var(\hat{F}_{10}(6)) = \frac{Var(N)}{100} = \frac{\cancel{10} \cdot q \cdot (1-q)}{\cancel{100}10} = \frac{q(1-q)}{10}$$

$$\therefore V_{10} = \frac{\hat{q}(1-\hat{q})}{10} = \frac{1}{40}$$

2. Determine a 95% linear symmetric confidence interval for the probability that a dragon dies within 6 years.
(Note that the 97.5 percentile of the standard normal distribution is 1.96.)

$$\hat{q} \pm 1.96\sqrt{V_{10}}$$

$$.5 \pm 1.96\sqrt{1/40}$$

$$\approx (0.19, 0.81)$$

Numbers 3, 4, and 5:
For a mortality table with a select period of two years, you are given:

x	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
50	0.050	0.065	0.080	52
51	0.055	0.070	0.085	53
52	0.060	0.075	0.090	54
53	0.065	0.080	0.095	55

3. Determine $1000\ {}_{3}p_{[52]}$

$$= 1000 \cdot p_{[52]} \cdot p_{[52]+1} \cdot p_{54} = 1000(.94)(.925)(.91) = 791.245$$

4. Using a uniform distribution of deaths assumption between integer ages, determine $1000_{1.5|0.5}q_{[50]}$

$$= 1000 \frac{l_{[50]+1.5} - l_{52}}{l_{[50]}} \underset{\text{below}}{\overset{\text{see values}}{=}} \cancel{26.125}\ \boxed{30.875}$$

Assume $l_{[50]} = 1000$

$$l_{[50]+1} = 1000 \cdot p_{[50]} = 1000(.95) = 950$$

$$l_{[50]+2} = l_{[50]+1} \cdot p_{[50]+1} = 950(\cancel{.946}\ 0.935) = \cancel{897.75}\ 888.25 = l_{52}$$

$$l_{[50]+1.5} \overset{UDD}{=} .5 \cdot l_{[50]+1} + .5 l_{52} = \cancel{923.875}\ 919.125$$

5. Using a constant force of mortality assumption between integer ages, determine $1000_{1|1.5}q_{[51]+0.5}$

$$= 1000 \frac{l_{[51]+1.5} - l_{54}}{l_{[51]+0.5}} \underset{\text{above}}{\overset{\text{see values}}{=}} 110.252\ldots$$

Assume $l_{[51]} = 1000$

$$l_{[51]+1} = 1000 \cdot p_{[51]} = 945$$

$$l_{[51]+2} = 945 \cdot p_{[51]+1} = 878.85 = l_{53}$$

$$l_{54} = l_{53} \cdot p_{53} = 878.85(.915) = 804.14\ldots$$

$$l_{[51]+0.5} \overset{CF}{=} l_{[51]}^{.5} \cdot l_{[51]+1}^{.5} = 972.11\ldots$$

$$l_{[51]+1.5} \overset{CF}{=} l_{[51]+1}^{.5} \cdot l_{53}^{.5} = 911.32\ldots$$