Show all work for full credit, and use correct notation. Simplify answers completely.

Numbers 1 and 2:
For a mortality study of 100 patients, you are given:

<table>
<thead>
<tr>
<th>Time Range in Year</th>
<th>Number of Deaths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 – 3</td>
<td>30</td>
</tr>
<tr>
<td>3 – 4</td>
<td>15</td>
</tr>
<tr>
<td>4 – 6</td>
<td>20</td>
</tr>
<tr>
<td>6 – 9</td>
<td>15</td>
</tr>
<tr>
<td>9+</td>
<td>20</td>
</tr>
</tbody>
</table>

1. Use the ogive to determine the probability that a randomly selected patient at the beginning of the study dies within 3.25 years.

\[
\Pr(T < 3) = \frac{30}{100} = 0.30 \text{ and } \Pr(T < 4) = \frac{30+15}{100} = 0.45
\]

\[
\therefore \Pr(T < 3.25) = 0.75(0.3) + 0.25(0.45) = 0.3375
\]

2. Use the ogive to determine the probability that a randomly selected patient at the beginning of the study survives 8 years.

\[
\Pr(T > 6) = \frac{15+20}{100} = 0.35 \text{ and } \Pr(T > 9) = \frac{30}{100} = 0.20
\]

\[
\therefore \Pr(T > 8) = \frac{1}{3}(0.35) + \frac{2}{3}(0.20) = 0.25
\]
Numbers 3 – 5:
For a mortality study on 10 dragons, you are given the following times for death:

\[
\begin{array}{cccccccccc}
2 & 3 & 3 & 6 & 8 & 12 & 18 & 18 & 18 & 23 \\
\end{array}
\]

3. Determine the empirical estimate for \( S(6) \), the probability that the time until death is greater than 6.

\[
p = S(6) \approx \hat{S}_{10}(6) = \frac{\text{number of observations greater than 6}}{10} = \frac{6}{10} = 0.6 \ (\hat{p})
\]

4. Approximate the variance of the empirical estimator for \( S(6) \).

The estimator is \( \hat{S}_{10}(6) = \frac{N}{10} \) where \( N \) is the random variable representing the number of the 10 observations that are greater than 6. Therefore \( N \sim Bin(m = 10, p) \), where \( p = \Pr (\text{a random dragon survives at least 6 years}) \).

\[
\therefore \text{Var} \left( \frac{\hat{S}_{10}(6)}{10} \right) = \text{Var} \left( \frac{N}{10} \right) = \frac{10 \cdot p \cdot (1-p)}{100} = \frac{p \cdot (1-p)}{10}
\]

The approximate variance is then \( V_{10} = \frac{\hat{p} \cdot (1-\hat{p})}{10} = \frac{0.6 \cdot 0.4}{10} = 0.024 \)

5. Determine an 80% linear symmetric confidence interval for \( S(6) \) based on the empirical estimator. Note the following percentiles of the standard normal distribution:

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.842</td>
</tr>
<tr>
<td>85</td>
<td>1.036</td>
</tr>
<tr>
<td>90</td>
<td>1.282</td>
</tr>
<tr>
<td>95</td>
<td>1.645</td>
</tr>
</tbody>
</table>

The 80% linear symmetric confidence interval for \( S(6) \) based on the empirical estimator is \( \hat{S}_{10}(6) \pm z_{0.9} \cdot \sqrt{V_{10}} = 0.6 \pm 1.282 \cdot \sqrt{0.024} = (0.4014, 0.7986) \).