MAP 4175 / 5177 Name:______________________________

Tests 5 Date: October 2, 2019

Show all work for full credit, and use correct notation. Simplify answers completely.

Numbers 1 and 2:

For a mortality study of 100 patients, you are given:

Time Range in Year	Number of Deaths
0 – 3	30
3 – 4	15
4 – 6	20
6 – 9	15
9+	20

1. Use the ogive to determine the probability that a randomly selected patient at the beginning of the study dies within 3.25 years.

$\Pr(T < 3) = \frac{30}{100} = 0.30$ and $\Pr(T < 4) = \frac{30+15}{100} = 0.45$

$$\therefore \Pr(T < 3.25) = 0.75(0.3) + 0.25(0.45) = 0.3375$$

2. Use the ogive to determine the probability that a randomly selected patient at the beginning of the study survives 8 years.

$\Pr(T > 6) = \frac{15+20}{100} = 0.35$ and $\Pr(T > 9) = \frac{30}{100} = 0.20$

$$\therefore \Pr(T > 8) = \frac{1}{3}(0.35) + \frac{2}{3}(0.20) = 0.25$$

Numbers 3 – 5:
For a mortality study on 10 dragons, you are given the following times for death:

2 3 3 6 8 12 18 18 18 23

3. Determine the empirical estimate for $S(6)$, the probability that the time until death is greater than 6.

$$p = S(6) \approx \hat{S}_{10}(6) = \frac{\text{number of observations greater than 6}}{10} = \frac{6}{10} = 0.6\ (= \hat{p})$$

4. Approximate the variance of the empirical estimator for $S(6)$.

The estimator is $\underline{\underline{\hat{S}}}_{10}(6) = \frac{N}{10}$ where N is the random variable representing the number of the 10 observations that are greater than 6. Therefore $N \sim Bin(m = 10, p)$, where $p = \Pr$ (a random dragon survives at least 6 years).

$$\therefore Var\left(\underline{\underline{\hat{S}}}_{10}(6)\right) = Var\left(\frac{N}{10}\right) = \frac{10 \cdot p \cdot (1-p)}{100} = \frac{p \cdot (1-p)}{10}$$

The approximate variance is then $V_{10} = \frac{\hat{p} \cdot (1-\hat{p})}{10} = \frac{0.6 \cdot 0.4}{10} = 0.024$

5. Determine an 80% linear symmetric confidence interval for $S(6)$ based on the empirical estimator. Note the following percentiles of the standard normal distribution:

Percentile	Value
80	0.842
85	1.036
90	1.282
95	1.645

The 80% linear symmetric confidence interval for $S(6)$ based on the empirical estimator is $\hat{S}_{10}(6) \pm z_{0.9} \cdot \sqrt{V_{10}} = 0.6 \pm 1.282 \cdot \sqrt{0.024} = (0.4014, 0.7986)$.