MAP 4175/5177 Take-Home Exam (Tests 6 & 7) KEY 10/17/18

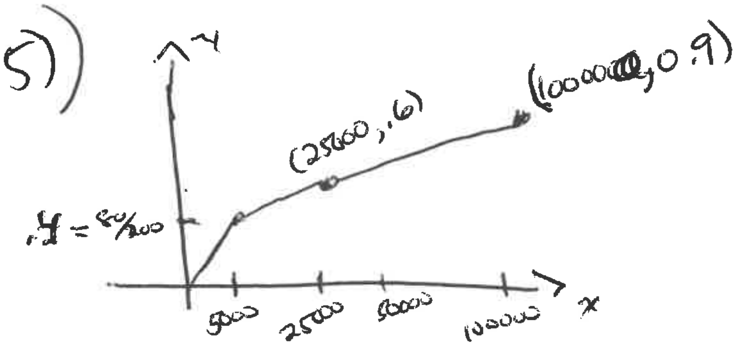
1) $\hat{S}(15) = \frac{4}{10} = \underline{0.4}$

2) $Var(\hat{\underline{\underline{S}}}(15)) \approx \frac{(.6)(.4)}{10} = \underline{0.024}$

3) $0.4 \pm 1.282\sqrt{0.024}$

$\underline{(0.201\cdots, 0.5986\cdots)}$

(4) & 5))



$F(5000) = 80/200 = 0.4$

$F(25000) = \frac{120}{200} = 0.6$

$F(100000) = 180/200 = 0.9$

4) $Pr(X < 100000) \approx F(100000) = \underline{0.9}$

5) $F(50000) \underset{\text{interpolation}}{\overset{\text{linear}}{=\!=}} \frac{2}{3}(.6) + \frac{1}{3}(.9) = 0.7$

$\therefore\ Pr(X > 50000) = 1 - Pr(X \leq 50000) \approx \underline{0.3}$

(5-9) $r_1 = 50$

$r_2 = 50-3-6 = 41$

$r_3 = 41-7-4 = 30$

$r_4 = 30-5-2 = 23$

$r_5 = 23-5-3 = 15$

$r_6 = 15-6-4 = 5$

6) $S_{50}(11) \overset{K-M}{=} \prod_{y_j \le 11} \left(1 - \frac{s_j}{r_j}\right) = \left(1-\frac{3}{50}\right)\left(1-\frac{7}{41}\right)\left(1-\frac{5}{30}\right)\left(1-\frac{5}{23}\right)$

$= \frac{47}{50} \cdot \frac{34}{41} \cdot \frac{25}{30} \cdot \frac{18}{23} = \underline{0.5083\ldots}$

$H_{50}(11) = -\ln(S_{50}(11)) = \underline{0.6765\ldots}$

7) $\hat{H}(11) \overset{N-A}{=} \sum_{y_j \le 11} \frac{s_j}{r_j} = \frac{3}{50} + \frac{7}{41} + \frac{5}{30} + \frac{5}{23} = \underline{0.6147\ldots}$

$\hat{S}(11) = e^{-\hat{H}(11)} = \underline{0.5407\ldots}$

8) (a) $Var(\underset{=}{S_{50}}(11)) \approx (0.5083\ldots)^2 \cdot \sum_{y_j \le 11} \frac{s_j}{r_j(r_j - s_j)}$

$= (0.5083\ldots)^2 \cdot \left(\frac{3}{50(47)} + \frac{7}{41(34)} + \frac{5}{30(25)} + \frac{5}{23(18)}\right) = \underline{0.00647\ldots}$

(b) $Var(\underset{=}{\hat{S}}(11)) \approx (0.5407\ldots)^2 \cdot \sum_{y_j \le 11} \frac{s_j(r_j - s_j)}{r_j^3}$

$= (0.5407\ldots)^2 \cdot \left(\frac{3(47)}{50^3} + \frac{7(34)}{41^3} + \frac{5(25)}{30^3} + \frac{5(18)}{23^3}\right) = \underline{0.00485\ldots}$

(c) $Var(\underset{=}{\hat{S}}(11)) = (0.5407\ldots)^2 \cdot \sum_{y_j \le 11} \frac{s_j}{r_j^2}$

$= (0.5407\ldots)^2 \cdot \left(\frac{3}{50^2} + \frac{7}{41^2} + \frac{5}{30^2} + \frac{5}{23^2}\right) = \underline{0.00595\ldots}$

9) (a) $S_{50}(25) = 0$

(b) $S_{50}(25) = S_{50}(20)$ since $\omega = 30$

$$S_{50}(20) \overset{K\text{-}M}{=} \underbrace{S_{50}(11)}_{\text{see \#6}} \cdot \left(1 - \frac{6}{15}\right)\left(1 - \frac{2}{5}\right)$$

$\therefore\ S_{50}(25) = 0.183\ldots$

(c) $S_{50}(25) = \left[\underbrace{S_{50}(20)}_{\text{see part (b)}}\right]^{25/20} = \underline{0.1197\ldots}$

10)

0	2	5	9	10
100 (Start) 6E 12W	4E 18W 18E 4W	12E 6W 18E 4W	12E 6W	Not Needed

8D, $s_1 = 8$ 20D, $s_2 = 20$ 30D, $s_3 = 30$

$r_1 = 100 + 6 - 12 = 94$

$r_2 = 94 - 8 + 4 - 18 + 18 - 4 = 86$

$r_3 = 86 - 20 + 12 - 6 + 18 - 4 = 86$

$$\therefore\ \hat{S}(9) \overset{K\text{-}M}{=} \left(1 - \frac{8}{94}\right)\left(1 - \frac{20}{86}\right)\left(1 - \frac{30}{86}\right) = 0.4571\ldots$$