MAP 4175 / 5177 Name:________________________________

Quiz 6 Date: October 7, 2015

Show all work for full credit, and use correct notation. Simplify answers completely. Unless told or implied otherwise, assume all lives are independent.

1. You are given:

   (i) Male mortality follows DeMoivre's Law with terminal age 100

   (ii) Female mortality follows a Constant Force model with $\mu = .02$

Determine ${}_{5|10}q_{20:30}$ where (20) is female and (30) is male.

$$
\begin{aligned}
{}_{5|10}q_{20:30} &= {}_{5}p_{20:30} - {}_{15}p_{20:30} \\
&= {}_{5}p_{20} \cdot {}_{5}p_{30} - {}_{15}p_{20} \cdot {}_{15}p_{30} \\
&= \left(e^{-5(.02)}\right)\left(\frac{100-30-5}{100-30}\right) - \left(e^{-15(.02)}\right)\left(\frac{100-30-15}{100-30}\right) \doteq .258
\end{aligned}
$$

2. Given ${}_{t}p_{\overline{xy}} = (1.05)^{-t}$, determine $e_{\overline{xy}:\overline{20|}}$

$$
\begin{aligned}
e_{\overline{xy}:\overline{20|}} &= p_{\overline{xy}} + {}_{2}p_{\overline{xy}} + {}_{3}p_{\overline{xy}} + \cdots + {}_{20}p_{\overline{xy}} \\
&= (1.05)^{-1} + (1.05)^{-2} + (1.05)^{-3} + \cdots + (1.05)^{-20} \\
&= v_{.05} + v_{.05}^{2} + v_{.05}^{3} + \cdots + v_{.05}^{20} = a_{\overline{20|}.05} = 12.462
\end{aligned}
$$

3. Mortality for smokers and non-smokers each follow a constant force model, but the force of mortality for smokers is twice the force of mortality for non-smokers. (x) is a smoker and (y) is a non-smoker. Given ${}_{10}q_{xy} = .6$, determine ${}_{30}p_{y}$.

Let μ = non-smoker force of mortality $\Rightarrow {}_{30}p_{y} = e^{-30\mu}$

Then 2μ = smoker force of mortality

$$
{}_{10}q_{xy} = .6 \Rightarrow {}_{10}p_{xy} = .4 \Rightarrow {}_{10}p_{x} \cdot {}_{10}p_{y} = .4
$$

$$
\Rightarrow e^{-10(2\mu)} \cdot e^{-10(\mu)} = .4 \Rightarrow e^{-30\mu} = .4
$$

$$
\therefore {}_{30}p_{y} = .4
$$

4. Mortality for non-smokers follows DeMoivre's Law with terminal age 100. Mortality for smokers follows Generalized DeMoivre's Law with $\alpha = 2$ and terminal age 90. Determine an expression for ${}_np_{40:50}$ where (40) is a smoker and (50) is a non-smoker.

$$
\begin{aligned}
{}_nP_{40:50} &= {}_nP_{40} \cdot {}_nP_{50} \\
&= \left(\frac{90-40-n}{90-40}\right)^2 \left(\frac{100-50-n}{100-50}\right) \\
&= \left(\frac{50-n}{50}\right)^2 \left(\frac{50-n}{50}\right) = \left(\frac{50-n}{50}\right)^3
\end{aligned}
$$

5. Given $q_{90} = .1$ and $q_{91} = .2$

(a) determine $e_{90:\overline{2}|}$

$$
\begin{aligned}
&= P_{90} + {}_2P_{90} \\
&= .9 + .9(.8) = 1.62
\end{aligned}
$$

(b) if $e_{92} = {}^{13}/_{9}$, determine e_{90}

$$
\begin{aligned}
e_{90} &= P_{90} + {}_2P_{90}(1 + e_{92}) \\
&= .9 + .9(.8)(1 + {}^{13}/_{9}) \\
&= 2.66
\end{aligned}
$$