MAP 4175 / 5177
Test 6

Name:_____

Date: October 12, 2016

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Given ${}_tp_{\overline{xy}} = e^{-.04t}$, determine $\mathring{e}_{\overline{xy}}$

$$\mathring{e}_{\overline{xy}} = \int_0^\infty {}_tP_{\overline{xy}}\, dt = \int_0^\infty e^{-.04t}\, dt = \frac{1}{.04} = 25$$

2. Determine the value of $T_{\overline{xy}}$ if $T_x + T_y = 40$ and $T_xT_y = 398.56$.

$$a+b=40 \quad a\cdot b = 398.56$$

$$b = 40-a$$

$$\therefore\ a(40-a) = 398.56 \Rightarrow a^2 - 40a + 398.56 = 0$$

$$\Rightarrow a = \frac{40 \pm \sqrt{(-40)^2 - 4(1)(+398.56)}}{2}$$

$$\Rightarrow a = 21.2 \text{ or } 18.8 = T_x$$

$$\Rightarrow T_y = 18.8 \text{ or } 21.2$$

$$T_{\overline{xy}} = \text{Max}(T_x, T_y) = 21.2$$

3. Given mortality for (40) follows a DML(90) model, determine $\overset{o}{e}_{40:\overline{10}|}$

$$\overset{o}{e}_{40:\overline{10}|} = \int_0^{10} {}_tP_{40}\,dt \qquad {}_tP_{40} = \frac{50-t}{50} = 1-.02t$$

$$\therefore \overset{o}{e}_{40:\overline{10}|} = \int_0^{10} (1-.02t)dt = 10 - .01(10)^2 = 9$$

(OR) $$\overset{o}{e}_{40:\overline{10}|} \overset{DML}{=\!=} 10 \cdot {}_{10}P_{40} + 5 \cdot {}_{10}\!\not{p}_{40}$$

$$= 10 \cdot \frac{40}{50} + 5 \cdot \frac{10}{50} = 9$$

4. Given ${}_tp_{xy} = (1.03)^{-t}$, determine $e_{xy:\overline{15}|}$

$$e_{xy:\overline{15}|} = P_{xy} + {}_2P_{xy} + \cdots + {}_{15}P_{xy}$$

$$= (1.03)^{-1} + (1.03)^{-2} + \cdots + (1.03)^{-15}$$

$$= a_{\overline{15}|.03} = 11.9379\ldots$$

5. Given $q_{80} = .10$ and $q_{81} = .11$ determine $e_{80:\overline{2}|}$

$$e_{80:\overline{2}|} = P_{80} + {}_2P_{80}$$

$$= .9 + .9(.89) = 1.701$$