MAP 4175 / 5177 Name:______________________________
Quiz 6 Date: October 18, 2017

Show all work for full credit, and use correct notation. Simplify answers completely. Unless told or implied otherwise, assume all lives are independent. See reverse side for additional problems. Each problem is worth 10 points

1. You are given:

   (i) The survival function for 20-year old males is ${}_tp_{20} = \left(\frac{60-t}{60}\right)^2$

   (ii) 40-year old females have a constant force of mortality of $\mu = 0.04$.

   (iii) ${}_{30}q_{20:\overset{2}{40}} = \int_0^{30} g(t)\ dt$ where (40) is female and (20) is male.

Determine $g(10)$.

$${}_{30}q_{20:\overset{2}{40}} = \int_0^{30} \underbrace{{}_tq_{20}}_{=1-\left(\frac{60-t}{60}\right)^2} \cdot \underbrace{{}_tp_{40}}_{=e^{-.04t}} \cdot \overbrace{\mu_{40}(t)}^{=.04}\, dt$$

$$\therefore\ g(t) = \left[1-\left(\frac{60-t}{60}\right)^2\right] \cdot e^{-.04t} \cdot (.04) \Rightarrow g(10) = 0.00819\ldots$$

2. You are given:

   (i) The survival function for 20-year old males is ${}_tp_{20} = \left(\frac{80-t}{80}\right)$ $\Rightarrow$ mortality for (20) is DML($\omega = 100$)

   (ii) Female mortality follows a constant force model with $\mu = 0.02$.

Determine ${}_5q^{\ 1}_{20:40}$ where (20) is male and (40) is female.

$${}_5q^{\ 1}_{20:40} = \int_0^5 \underbrace{{}_tp_{40}}_{=e^{-.02t}} \cdot \underbrace{{}_tp_{20} \cdot \mu_{20}(t)}_{=\frac{1}{80}}\, dt$$

$$\therefore\ {}_5q^{\ 1}_{20:40} = \frac{1}{80}\int_0^5 e^{-.02t}\, dt$$

$$= \frac{1}{80}\cdot\frac{1}{.02}\, e^{-.02t}\Big|_5^0 = \frac{1}{1.6}\left(1-e^{-.1}\right)$$

$$= .0594\ldots$$

3. The force of mortality for smokers is $\mu = 0.2$, and the force of mortality for non-smokers is $\mu = 0.1$.

Determine the difference $\underbrace{{}_{10}q_{x:y}^{\;\,1} - {}_{10}q_{x:y}^{2}}$ where (x) is a smoker and (y) is a non-smoker.

$$\Delta = {}_{10}q_y \cdot {}_{10}p_x = (1 - e^{-10(.1)}) \cdot e^{-10(.2)}$$

$$= .0855\ldots$$

4. For a 25-year old male and 30-year old female, you are given:

(i) $\mu_{25}(t) = 0.2$ for $0 \le t \le 10$

(ii) ${}_tq_{30} = 0.01t$ for $0 \le t \le 10$ $\Longrightarrow$ mortality for (30) over next 10 years has a uniform distribution of deaths

Determine the probability that within the next 5 years, the 30-year old female will die after a 25-year old male.

$${}_5q_{25:30}^{\;\;\;\,2} = \int_0^5 {}_tq_{25} \cdot \underbrace{{}_tp_{30} \cdot \mu_{30}(t)}_{=.01}\, dt$$

$$\therefore\ {}_5q_{25:30}^{\;\;\;\,2} = .01 \int_0^5 (1 - e^{-.2t})\, dt = .01\left(t + \tfrac{1}{.2} e^{-.2t}\right)\Big|_0^5$$

$$= .01\left(5 + \tfrac{1}{.2} e^{-1}\right) - .01\left(0 + \tfrac{1}{.2}\right) = .01839\ldots$$

5. Given a smoker whose force of mortality is $\mu^s = 0.3$, and a non-smoker whose force of mortality is $\mu^{ns} = 0.1$, determine the probability that the smoker dies first.

$${}_\infty q_{s:ns}^{1} = \int_0^\infty \underbrace{{}_tp_x^{ns}}_{=e^{-.1t}} \cdot \underbrace{{}_tp_y^{s}}_{=e^{-.3t}} \cdot \underbrace{\mu_y^{s}(t)}_{=.3}\, dt$$

$$\therefore\ {}_\infty q_{s:ns}^{1} = .3 \int_0^\infty e^{-.4t}\, dt = \frac{.3}{.4} \cdot e^{-.4t}\Big|_\infty^0 = \frac{3}{4}$$

Remark: $${}_\infty q_{s:ns}^{1} \overset{CF}{=} \frac{\mu^s}{\mu^s + \mu^{ns}} = \frac{.3}{.4} = \frac{3}{4}$$