MAP 4175 / 5177 Name:________________________________

Quiz 7 Date: October 14, 2015

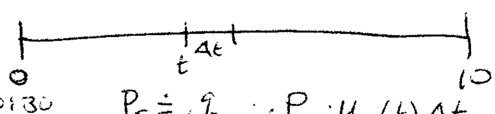
Show all work for full credit, and use correct notation. Simplify answers completely. Unless told or implied otherwise, assume all lives are independent.

1. You are given:

   (i) Male mortality follows DeMoivre's Law with terminal age 80

   (ii) Female mortality follows DeMoivre's Law with terminal age 100

Determine ${}_{10}q_{30:\overset{2}{20}}$ where (20) is female and (30) is male.



$\Pr \doteq {}_tq_{30} \cdot {}_tp_{20} \cdot \mu_{20}(t)\,\Delta t$

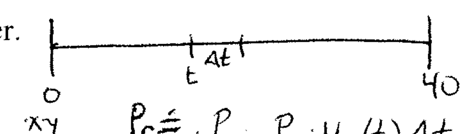
$$\therefore \quad {}_{10}q_{30:\overset{2}{20}} = \int_0^{10} \underbrace{{}_tq_{30}}_{=\frac{t}{80-30}=\frac{t}{50}} \cdot \underbrace{{}_tp_{20} \cdot \mu_{20}(t)}_{=\frac{1}{100-20}=\frac{1}{80}=f_{20}}\,dt$$

$$\therefore \quad {}_{10}q_{30:\overset{2}{20}} = \int_0^{10} \frac{t}{50}\cdot\frac{1}{80}\,dt = \frac{1}{4000}\int_0^{10} t\,dt = \frac{1}{4000}\cdot\frac{t^2}{2}\Big|_0^{10}$$

$$= \frac{50}{4000} = .0125$$

2. The force of mortality for smokers is $\mu = .04$, and the force of mortality for non-smokers is $\mu = .02$.

Determine ${}_{40}q^{1}_{x:y}$ where (x) is a smoker and (y) is a non-smoker.



$\Pr \doteq {}_tp_y \cdot {}_tp_x \cdot \mu_x(t)\,\Delta t$

$${}_{40}q^{1}_{xy} = \int_0^{40} {}_tp_y \cdot {}_tp_x \cdot \mu_x(t)\,dt$$

$$= \int_0^{40} e^{-.02t} \cdot e^{-.04t} \cdot (.04)\,dt$$

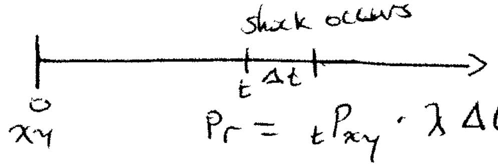
$$= .04\int_0^{40} e^{-.06t}\,dt = \frac{.04}{.06}\,e^{-.06t}\Big|_{40}^{0} = \frac{2}{3}\left(1-e^{-2.4}\right)$$

3. Given ${}_nq_{x:\overset{2}{y}} = .02$ and ${}_nq^{2}_{x:y} = .03$, determine ${}_np_{\overline{xy}}$

$${}_nq_{\overline{xy}} = {}_nq_{x\overset{2}{y}} + {}_nq^{2}_{xy} = .02 + .03 = .05$$

$$\Longrightarrow \quad {}_np_{\overline{xy}} = .95$$

4. For a common shock model with $\lambda = .001$, in the absence of the shock the future lifetimes of (x) and (y) follow constant force models with $\mu_x^* = .010$ and $\mu_y^* = .009$. Determine the probability that $\underbrace{\text{both } (x) \text{ and } (y) \text{ die at the same time}}_{\text{shock occurs}}$.



$Pr = {}_tp_{xy} \cdot \lambda \Delta t$

$${}_tp_{xy} = e^{-(.01+.009+.001)t} = e^{-.02t}$$

$$\therefore Pr(T_x = T_y) = \int_0^\infty {}_tp_{xy} \cdot \lambda \, dt = .001 \int_0^\infty e^{-.02t} \, dt$$

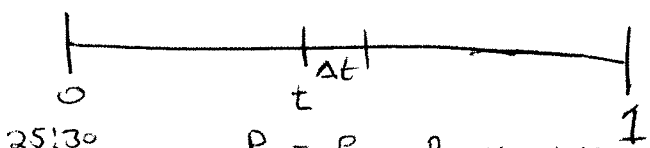
$$= \frac{.001}{.02} = .05$$

5. For 25-year old males and 30-year old females, you are given:

(i) $\mu_{25}(t) = .2$ for $0 \le t \le 1$

(ii) ${}_tq_{30} = .01t$ for $0 \le t \le 1$ (deaths are uniformly distributed for females)

Determine the probability that a 30-year old female will die before a 25-year old male and within the next year.



$Pr = {}_tp_{25} \cdot {}_tp_{30} \cdot \mu_{30}(t) \Delta t$

$$\therefore P = q_{25:\overset{1}{30}} = \int_0^1 {}_tp_{25} \cdot \underbrace{{}_tp_{30} \cdot \mu_{30}(t)}_{\overset{UDD}{=} q_{30} = .01} \, dt$$

$$\therefore q_{25:\overset{1}{30}} = .01 \int_0^1 e^{-.2t} \, dt$$

$$= \frac{.01}{.2} e^{-.2t} \Big|_1^0 = .05\left(1 - e^{-.2}\right)$$