MAP 4175 / 5177 Name:________________________________

Quiz 7 Date: October 26, 2016

Show all work for full credit, and use correct notation. Simplify answers completely. Unless told or implied otherwise, assume all lives are independent. See reverse side for additional problems.

1. You are given:

   (i) The survival function for 20-year old males is ${}_tp_{20} = \left(\frac{60-t}{60}\right)^2$ $\Rightarrow GDML(\alpha=2, \omega=80)$

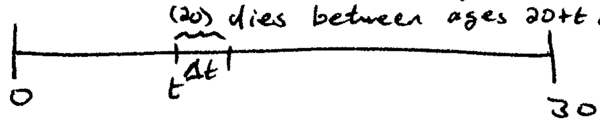
   (ii) 40-year old females have a constant force of mortality of $\mu = 0.04$.

   (iii) ${}_{30}q^{\,1}_{20:40} = \int_0^{30} g(t)\ dt$ where (40) is female and (20) is male.

Determine $g(10)$.

(40) survives to age 40+t, and

(20) dies between ages 20+t and 20+t+Δt



20:40

$Pr = {}_tP_{40} \cdot {}_tP_{20} \cdot \mu_{20+t} \cdot \Delta t$

$$g(t) = {}_tP_{40} \cdot {}_tP_{20} \cdot \mu_{20}(t)$$

$$= e^{-.04t} \cdot \left(\frac{60-t}{60}\right)^2 \cdot \frac{2}{60-t} \Rightarrow g(10) = e^{-.4} \cdot \left(\frac{50}{60}\right)^2 \cdot \frac{2}{50} = .01862$$

2. You are given:

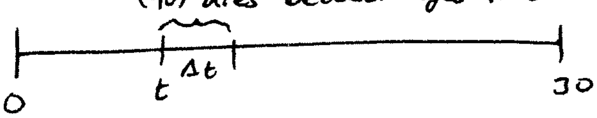
   (i) The survival function for 20-year old males is ${}_tp_{20} = \left(\frac{60-t}{60}\right)^2$

   (ii) Newborn female mortality follows a uniform distribution over the interval (0,100)

Determine ${}_{30}q^{\,2}_{40:20}$ where (40) is female and (20) is male.

(20) dies before age 20+t, and

(40) dies between ages 40+t and 40+t+Δt



40:20

$Pr = {}_tq_{20} \cdot {}_tP_{40} \cdot \mu_{40}(t) \cdot \Delta t$

$${}_{30}q^{\,2}_{40:20} = \int_0^{30} {}_tq_{20} \cdot {}_tP_{40} \cdot \mu_{40}(t) \cdot dt \qquad {}_tP_{40} \cdot \mu_{40}(t) \underset{\omega=100}{\overset{DML}{=}} q_{40} = \frac{1}{60}$$

$$= \int_0^{30} \left(1 - \left(\frac{60-t}{60}\right)^2\right) \cdot \frac{1}{60}\ dt$$

$$= \frac{1}{60}\left[30 - \left(\frac{1}{3}\left(\frac{60-t}{60}\right)^3(+60)\right)\Big|_{30}^{0}\right]$$

$$= \frac{1}{60}\left[30 - \left(20 - 20\left(\frac{1}{2}\right)^3\right)\right] = .2083$$

3. The force of mortality for smokers is $\mu = 2$, and the force of mortality for non-smokers is $\mu = 1$.

Determine the difference ${}_2q_{x:\overset{1}{y}} - {}_2q^{\,2}_{x:y}$ where (x) is a smoker and (y) is a non-smoker.

$$= {}_2q_y \cdot {}_2p_x = (1 - e^{-2\cdot 1}) \cdot e^{-2\cdot 2}$$

$$= (1 - e^{-2}) \cdot e^{-4} = .0158$$

4. For 25-year old males and 30-year old females, you are given:

(i) $\mu_{25}(t) = 0.2$ for $0 \le t \le 10$

(ii) ${}_tq_{30} = 0.01t$ for $0 \le t \le 10$

Determine the probability that a 30-year old female will die before a 25-year old male and within the next five years.

(25) lives to age 25+t
(30) dies between ages 30+t and 30+t+Δt

0   t Δt 5

25:30 $Pr = {}_tp_{25} \cdot {}_tp_{30} \cdot \mu_{30}(t) \cdot \Delta t$

$$\therefore P = {}_5q_{25:\overset{1}{30}} = \int_0^5 \underbrace{{}_tp_{25}}_{= e^{-.2t}} \cdot \underbrace{{}_tp_{30} \cdot \mu_{30}(t)}_{= (1-.01t)\cdot \frac{.01}{1-.01t} = .01} \, dt$$

$$= .01 \int_0^5 e^{-.2t} \, dt = \frac{.01}{.2} e^{-.2t} \Big|_5^0 = \frac{1}{20}(1 - e^{-1}) = .0316$$

5. For a common shock model with $\lambda = 0.01$, in the absence of the shock the future lifetimes of (x) and (y) follow constant force models with $\mu_x^* = 0.01$ and $\mu_y^* = 0.08$. Determine the probability that (x) dies before (y).

both (x) and (y) survive t years, and (x+t) dies but (y+t) does not

Note: ${}_tp_{xy} = e^{-(.01+.08+.01)t} = e^{-.1t}$

0   t Δt

xy $Pr = {}_tp_{xy} \cdot \mu_x^*(t) \cdot \Delta t$

$$\therefore P = {}_\infty q_{\overset{1}{x}y} = Pr(T_x < T_y) = \int_0^\infty e^{-.1t}(.01)\, dt$$

$$= \frac{.01}{.1} e^{-.1t} \Big|_\infty^0 = \frac{1}{10}$$