MAP 4175 / 5177 Name:______________________________

Test 7 Date: October 25, 2017

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. For a double decrement table, given $q_x'^{(1)} = 0.3$, $p_x'^{(2)} = 0.8$ and $q_x^{(2)} = 0.18$, determine $q_x^{(1)}$.

$p_x'^{(1)} = .7 \quad \therefore p_x^{(\tau)} = .7(.8) = .56$

$\therefore q_x^{(\tau)} = 1 - .56 = .44 = q_x^{(1)} + q_x^{(2)} = q_x^{(1)} + 0.18$

$\Rightarrow q_x^{(1)} = 0.26$

2. You are given the double decrement table, where decrement d refers to death and decrement w refers to withdrawal:

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w)}$
50	1000	-	200
51	-	-	50
52	300	100	-

Determine

(a) ${}_2p_{50}^{(\tau)}$

(b) ${}_{2|}q_{50}^{(d)}$

(a) ${}_2p_{50}^{(\tau)} = \dfrac{l_{52}^{(\tau)}}{l_{50}^{(\tau)}} = \dfrac{300}{1000} = 0.3$

(b) ${}_{2|}q_{50}^{(d)} = \dfrac{d_{52}^{(d)}}{l_{50}^{(\tau)}} = \dfrac{100}{1000} = 0.1$

3. You are given the double decrement table, where decrement d refers to death and decrement w refers to withdrawal:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$q_x^{(1)}$	$d_x^{(2)}$	$q_x^{(2)}$
95	-	600	-	-	0.10
96	- 3000	- 1200	0.40	-	0.20
97	- 1200	- 900	0.75	300	0.25

Determine ${}_{1|2}q_{95}^{(1)}$. $= \dfrac{2d_{96}^{(1)}}{l_{95}^{(\tau)}}$

Since $q_{97}^{(1)} = 3\cdot q_{97}^{(2)}$, then $d_{97}^{(1)} = 3\cdot d_{97}^{(2)} = 900$. $\therefore l_{97}^{(\tau)} = 1200$

$q_{96}^{(\tau)} = 0.4+0.2=0.6 \Rightarrow p_{96}^{(\tau)} = 0.4 \Rightarrow l_{96}^{(\tau)} = \dfrac{l_{97}}{p_{96}^{(\tau)}} = \dfrac{1200}{.4} = 3000$

$d_{96}^{(1)} = 3000(.4) = 1200$

$l_{95}^{(\tau)} - 600 - d_{95}^{(2)} = l_{96}^{(\tau)}$ $\quad d_{95}^{(\tau)} = l_{95}^{(\tau)}(.1)$ $\quad l_{96}^{(\tau)} = 3000$

$\therefore l_{95}^{(\tau)} - 600 - .1 l_{95}^{(\tau)} = 3000 \Rightarrow l_{95}^{(\tau)} = 4000$

$\therefore {}_{1|2}q_{95}^{(1)} = \dfrac{1200+900}{4000} = 0.525$

4. For a triple decrement table, given $\mu_x^{(1)}(t) = 0.15$, $\mu_x^{(2)}(t) = 0.20$, and $\mu_x^{(3)}(t) = 0.65$ determine ${}_{0.3|0.5}q_x^{(3)}$.

$\mu_x^{(\tau)}(t) = .15+.2+.65 = 1 \Rightarrow {}_tp_x^{(\tau)} = e^{-1\cdot t} = e^{-t}$

Since $\dfrac{\mu_x^{(3)}(t)}{\mu_x^{(\tau)}(t)} = .65$, then ${}_{.3|.5}q_x^{(3)} = .65\cdot {}_{.3|.5}q_x^{(\tau)}$

${}_{.3|.5}q_x^{(\tau)} = {}_{.3}p_x^{(\tau)} - {}_{.8}p_x^{(\tau)}$

$= e^{-.30} - e^{-.80}$

$\therefore {}_{.3|.5}q_x^{(3)} = .65(e^{-.3} - e^{-.8}) = .1894\ldots$