MAP 4175 / 5177 Name:______________________________

Tests 7 Date: October 16, 2019

Show all work for full credit, and use correct notation. Simplify answers completely.

Numbers 1 and 2 use the following data:
In a mortality study on 100 dragons, all age 50, you are given:

I. Between ages 50 and 51,
   20 dragons die
   30 dragons fly away
   10 dragons enter the study
II. Between ages 51 and 52,
   30 dragons die
   10 dragons fly away
   20 dragons enter the study

1. Using the Kaplan-Meier approach, determine r_1, the size of the risk set at the first time at which dragons die, and r_2, the size of the risk set at the second time at which dragons die.

$$r_1 = 100 - 15(W) + 5(E) = 90$$

$$r_2 = 90 - 20(D) - 15(W) + 5(E) - 5(W) + 10(E) = 65$$

2. Using the Kaplan-Meier approach, determine the approximate value of $S(2)$, the probability that a randomly selected dragon in the study survives at least two years.

$$S(2) \approx \prod_{y_j \le 2} \left(1 - \frac{s_j}{r_j}\right) = \left(1 - \frac{20}{90}\right) \cdot \left(1 - \frac{30}{65}\right) = \frac{7}{9} \cdot \frac{35}{65} = \frac{49}{117}$$

3. Determine the value of T_{xy} if $T_x + T_y = 67.7$ and $T_x T_y = 830.5$

In order to simplify notation, let $a = T_x$ and $b = T_y$.

Then we have $a + b = 67.7$ and $a \cdot b = 830.5$. Using substitution, we get
$a \cdot (67.7 - a) = 830.5 \;\; \Rightarrow \;\; a^2 - 67.7a + 830.5 = 0$

Solving the quadratic gives $a = T_x = 51.6\cdots$ and $b = T_y = 16.0\cdots$, or
$a = T_x = 16.0\cdots$ and $b = T_y = 51.6\cdots$

Then $T_{xy} = Min(T_x, T_y) = 16.0\cdots$

4. Given ${}_{20}q_x = {}_{20}q_y = 0.4$ and ${}_{20}q_{xy} = 0.64$, determine ${}_{20}p_{\overline{xy}}$

${}_{20}q_{\overline{xy}} = {}_{20}q_x + {}_{20}q_y - {}_{20}q_{xy} = 0.4 + 0.4 - 0.64 = 0.16$

$$\therefore \; {}_{20}p_{\overline{xy}} = 0.84$$

Alternatively

$${}_{20}p_{\overline{xy}} = {}_{20}p_x + {}_{20}p_y - {}_{20}p_{xy} = 0.6 + 0.6 - 0.36 = 0.84$$

5. Given ${}_{t}p_{xy} = 0.92^t$, determine e_{xy}.

$$e_{xy} = p_{xy} + {}_{2}p_{xy} + {}_{3}p_{xy} + \cdots = 0.92 + 0.92^2 + 0.92^3 + \cdots = \frac{0.92}{1 - 0.92} = 11.5$$