Show all work for full credit, and use correct notation. Simplify answers completely.

Numbers 1 and 2 use the following data:
In a mortality study on 100 dragons, all age 50, you are given:

I. Between ages 50 and 51,
   - 20 dragons die
   - 30 dragons fly away
   - 10 dragons enter the study
II. Between ages 51 and 52,
    - 30 dragons die
    - 10 dragons fly away
    - 20 dragons enter the study

1. Using the Kaplan-Meier approach, determine \( r_1 \), the size of the risk set at the first time at which dragons die, and \( r_2 \), the size of the risk set at the second time at which dragons die.

\[
r_1 = 100 - 15(W) + 5(E) = 90
\]
\[
r_2 = 90 - 20(D) - 15(W) + 5(E) - 5(W) + 10(E) = 65
\]

2. Using the Kaplan-Meier approach, determine the approximate value of \( S(2) \), the probability that a randomly selected dragon in the study survives at least two years.

\[
S(2) \approx \prod_{y \leq 2} \left(1 - \frac{s_j}{r_j}\right) = \left(1 - \frac{20}{90}\right) \cdot \left(1 - \frac{30}{65}\right) = \frac{7}{9} \cdot \frac{35}{65} = \frac{49}{117}
\]
3. Determine the value of $T_{xy}$ if $T_x + T_y = 67.7$ and $T_x T_y = 830.5$

In order to simplify notation, let $a = T_x$ and $b = T_y$.

Then we have $a + b = 67.7$ and $a \cdot b = 830.5$. Using substitution, we get $a \cdot (67.7 - a) = 830.5 \Rightarrow a^2 - 67.7a + 830.5 = 0$

Solving the quadratic gives $a = T_x = 51.6 \ldots$ and $b = T_y = 16.0 \ldots$, or $a = T_x = 16.0 \ldots$ and $b = T_y = 51.6 \ldots$

Then $T_{xy} = Min(T_x, T_y) = 16.0 \ldots$

4. Given $20q_x = 20q_y = 0.4$ and $20q_{xy} = 0.64$, determine $20p_{xy}$

$20q_{xy} = 20q_x + 20q_y - 20q_{xy} = 0.4 + 0.4 - 0.64 = 0.16$

$\therefore 20p_{xy} = 0.84$

Alternatively

$20p_{xy} = 20p_x + 20p_y - 20p_{xy} = 0.6 + 0.6 - 0.36 = 0.84$

5. Given $p_{xy} = 0.92^t$, determine $e_{xy}$.

$e_{xy} = p_{xy} + 2p_{xy}^2 + 3p_{xy}^3 + \cdots = 0.92 + 0.92^2 + 0.92^3 + \cdots = \frac{0.92}{1 - 0.92} = 11.5$