MAP 4175 / 5177 Name:____________________________

Quiz 8 Date: October 21, 2015

Show all work for full credit, and use correct notation. Simplify answers completely.

1. For a double decrement table, given $p_x'^{(1)} = 0.6$ and $q_x^{(\tau)} = 0.52$, determine $q_x'^{(2)}$

$$p_x^{(\tau)} = p_x'^{(1)} \cdot p_x'^{(2)} \implies 0.48 = 0.6 \cdot p_x'^{(2)}$$

$$\implies p_x'^{(2)} = .8 \implies q_x'^{(2)} = .2$$

2. You are given the double decrement table, where decrement d refers to death and decrement w refers to withdrawal:

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w)}$
50	1000	20	35
51	945	25	25
52	895	30	0

Determine

(a) ${}_3p_{50}^{(\tau)} = \dfrac{l_{53}^{(\tau)}}{l_{50}^{(\tau)}} = \dfrac{895-30}{1000} = .865$

(b) ${}_{1|}q_{50}^{(d)} = \dfrac{d_{51}^{(d)}}{l_{50}^{(\tau)}} = \dfrac{25}{1000} = .025$

3. You are given the double decrement table:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$q_x^{(1)}$	$d_x^{(2)}$	$q_x^{(2)}$
95	-	150	-	-	0.15
96	- 360	-	0.40	-	0.40
97	- 72	- 36	0.50	36	0.50
98	0				

Determine $l_{95}^{(\tau)}$

$$d_x^{(1)} = d_x^{(2)} = 36 \quad \text{since} \quad q_x^{(1)} = q_x^{(2)}$$

$$\therefore\ l_{97}^{(\tau)} = 36 + 36 = 72$$

$$q_{96}^{(\tau)} = q_{96}^{(1)} + q_{96}^{(2)} = .8 \Rightarrow p_{96}^{(\tau)} = .2 \Rightarrow l_{96}^{(\tau)} = \frac{72}{.2} = 360$$

$$l_{95}^{(\tau)} - 150 - d_{95}^{(2)} = 360 \qquad d_{95}^{(2)} = l_{95}^{(\tau)} \cdot q_{95}^{(2)} = .15\, l_{95}^{(\tau)}$$

$$\therefore\ l_{95}^{(\tau)} - 150 - .15\, l_{95}^{(\tau)} = 360$$

$$\Longrightarrow l_{95}^{(\tau)} = 600$$

4. For a triple decrement table, given $\mu_x^{(1)}(t) = .05t$, $\mu_x^{(2)}(t) = .20t$, and $\mu_x^{(3)}(t) = .75t$ determine ${}_{0.5|0.3}q_x^{(3)}$

$$\mu_{x+t}^{(\tau)} = .05t + .2t + .75t$$

$$\Rightarrow \mu_{x+t}^{(\tau)} = t$$

$$_{.5|.3}q_x^{(3)} = \int_{.5}^{.8} {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(3)}\, dt$$

$$_np_x^{(\tau)} = e^{-\int_0^n \mu_{x+t}^{(\tau)} dt} = e^{-\int_0^n t\, dt} = e^{-n^2/2}$$

$$\therefore \int_{.5}^{.8} {}_tp_x^{(\tau)} \cdot \mu_{x+t}^{(3)}\, dt = \int_{.5}^{.8} e^{-t^2/2} \cdot (.75t)\, dt$$

t	u
.5	.125
.8	.32

$$u = t^2/2$$

$$du = t\, dt$$

$$= \int_{.125}^{.32} e^{-u} \cdot (.75)\, du$$

$$= .75\, e^{-u} \Big|_{.32}^{.125} = .75\left(e^{-.125} - e^{-.32}\right)$$

$$\therefore\ _{.5|.3}q_x^{(3)} = .75\left(e^{-.125} - e^{-.32}\right)$$