MAP 4175 / 5177 Name:________________________

Test 8 Date: November 2, 2016

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. For a double decrement table, given $\underbrace{p_x'^{(1)} = 0.6,\ p_x'^{(2)} = 0.9}$ and $q_x^{(1)} = 0.38$, determine $q_x^{(2)}$.

$$\Rightarrow p_x^{(\tau)} = (.6)(.9) = .54 \Rightarrow q_x^{(\tau)} = .46$$

$$\Rightarrow q_x^{(2)} = .46 - .38 = .08$$

2. You are given the double decrement table, where decrement *d* refers to death and decrement *w* refers to withdrawal:

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w)}$
50	1000	30	50
51	- 920	- 20	20
52	880	30	0
53	850		

Determine

(a) ${}_3p_{50}^{(\tau)}$

(b) ${}_{1|}q_{50}^{(d)}$

(a) $${}_3p_{50}^{(\tau)} = \frac{l_{53}^{(\tau)}}{l_{50}^{(\tau)}} = \frac{850}{1000}$$

(b) $${}_{1|}q_{50}^{(d)} = \frac{d_{51}^{(d)}}{l_{50}^{(\tau)}} = \frac{20}{1000}$$

3. You are given the double decrement table, where decrement d refers to death and decrement w refers to withdrawal:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$q_x^{(1)}$	$d_x^{(2)}$	$q_x^{(2)}$
95	-	60	-	- 540	0.15
96	-3000	- 1200	0.40	- 1200	0.40
97	- 600	- 300	0.50	300	0.50

Determine ${}_2q_{95}^{(2)}$.

$$q_{96}^{(\tau)} = q_{96}^{(1)} + q_{96}^{(2)} = .8 \Rightarrow p_{96}^{(\tau)} = .2$$

$${}_2q_{95}^{(2)} = \frac{{}_2d_{95}^{(2)}}{l_{95}^{(\tau)}}$$

$$\therefore l_{96}^{(\tau)} = \frac{l_{97}^{(\tau)}}{p_{96}^{(\tau)}} = \frac{600}{.2} = 3000 \Rightarrow d_{96}^{(2)} = 3000(.4) = 1200$$

$$l_{95}^{(\tau)} - 60 - .15\, l_{95}^{(\tau)} = 3000 \Rightarrow l_{95}^{(\tau)} = 3600$$

$$d_{95}^{(2)} = 3600 \cdot (.15) = 540$$

$$\therefore {}_2q_{95}^{(2)} = \frac{540 + 1200}{3600} = \frac{1740}{3600} = 0.483\cdots$$

4. For a triple decrement table, given $\mu_x^{(1)}(t) = .05t$, $\mu_x^{(2)}(t) = .20t$, and $\mu_x^{(3)}(t) = .55t$ determine ${}_{0.3|0.5}q_x^{(2)}$.

$${}_{.3|.5}q_x^{(2)} = \int_{.3}^{.8} {}_tp_x^{(\tau)} \cdot \mu_x^{(2)}(t)\, dt$$

$$\mu_x^{(\tau)}(t) = .05t + .2t + .55t = .8t \Rightarrow \mu_x^{(2)}(t) = \frac{1}{4} \cdot \mu_x^{(\tau)}(t)$$

$$\therefore {}_{.3|.5}q_x^{(2)} = \frac{1}{4} \cdot {}_{.3|.5}q_x^{(\tau)} = \frac{1}{4}\left[{}_{.3}p_x^{(\tau)} - {}_{.8}p_x^{(\tau)} \right]$$

$${}_np_x^{(\tau)} = e^{-\int_0^n \mu_x^{(\tau)}(t)\,dt} = e^{-\int_0^n .8t\,dt} = e^{-.4n^2}$$

$$\therefore {}_{.3|.5}q_x^{(2)} = \frac{1}{4}\left[e^{-.4(.3)^2} - e^{-.4(.8)^2} \right] = \frac{1}{4}\left[e^{-.036} - e^{-.256} \right]$$

$$= .0476\cdots$$