MAP 4175 / 5177 Name:____________________

Quiz 8 – Take Home Date: November 1, 2017 – Due on November 2, 2017

Show all work for full credit, and use correct notation. Simplify answers completely.

1. Given a triple decrement model with $\mu_x^{(1)} = 0.1$, $\mu_x^{(2)} = 0.2$, and $\mu_x^{(3)} = 0.3$ determine ${}_{0.25}q_x^{(2)}$.

$${}_{.25}q_x^{(2)} = \frac{1}{3} \cdot {}_{.25}q_x^{(\tau)} \quad \text{since} \quad \frac{\mu_x^{(2)}}{\mu_x^{(\tau)}} = \frac{.2}{.6} = \frac{1}{3}$$

$${}_n p_x^{(\tau)} = {}_n p_x^{\prime(1)} \cdot {}_n p_x^{\prime(2)} \cdot {}_n p_x^{\prime(3)} = e^{-.1n} \cdot e^{-.2n} \cdot e^{-.3n} = e^{-.6n}$$

$$\therefore \; {}_{.25}p_x^{(\tau)} = e^{-.6(.25)} = e^{-.15}$$

$$\Rightarrow \; {}_{.25}q_x^{(2)} = \frac{1}{3} \cdot (1 - e^{-.15}) = 0.046\cdots$$

2. For a double decrement model where each decrement is uniformly distributed in the double decrement table, given $q_x^{\prime(1)} = 0.20$ and $q_x^{\prime(2)} = 0.40$, determine ${}_{0.5}p_x^{\prime(2)}$.

MUDD

$${}_t p_x^{\prime(2)} = \left[{}_t p_x^{(\tau)}\right]^{\left(\frac{q_x^{(2)}}{q_x^{(\tau)}}\right)} \qquad p_x^{\prime(1)} = .8 \qquad p_x^{\prime(2)} = .6 \qquad \therefore \; p_x^{(\tau)} = .48$$

$$q_x^{(\tau)} = .52$$

$$t = .5 \Rightarrow {}_{.5}p_x^{\prime(2)} = \left[{}_{.5}p_x^{(\tau)}\right]^{\left(\frac{q_x^{(2)}}{q_x^{(\tau)}}\right)}$$

$${}_{.5}p_x^{(\tau)} = 1 - {}_{.5}q_x^{(\tau)} \overset{\text{MUDD}}{=\!=} 1 - .5 \cdot q_x^{(\tau)} = .74$$

We need $q_x^{(2)}$! Use the MUDD formula with $t = 1$

$$t = 1: \quad .6 = [.48]^{\left(q_x^{(2)}/.52\right)} \Rightarrow q_x^{(2)} = .52 \cdot \frac{\ln(.6)}{\ln(.48)} = .3619\ldots$$

$$\therefore \; {}_{.5}p_x^{\prime(2)} = [.74]^{\left(\frac{.3619\ldots}{.52}\right)} = 0.8109\ldots$$

3. For a double decrement model, you are given:

(i) in the associated single decrement model, decrement 1 is uniformly distributed with a terminal age of 100 $\Rightarrow {}_n p_x^{\prime(1)} = \frac{100-x-n}{100-x}$

(ii) decrement 2 has a constant force of departure equal to 0.02 $\Rightarrow {}_n p_x^{\prime(2)} = e^{-.02n}$

Determine ${}_{10}q_{75}^{(2)}$.

$$(i) \Rightarrow {}_t p_{75}^{\prime(1)} \cdot \mu_{75+t}^{(1)} = q_{75}^{\prime(1)} = \frac{1}{100-75} = .04$$

Since (1) is SUPD, calculate ${}_{10}q_{75}^{(1)}$ first, and then ${}_{10}q_{75}^{(2)} = {}_{10}q_{75}^{(\tau)} - {}_{10}q_{75}^{(1)}$

$${}_{10}q_{75}^{(1)} = \int_0^{10} {}_t p_{75}^{\prime(2)} \cdot \underbrace{{}_t p_{75}^{\prime(1)} \cdot \mu_{75+t}^{(1)}}_{=.04} \, dt$$

$$= .04 \int_0^{10} e^{-.02t} \, dt = \frac{.04}{.02} e^{-.02t} \Big|_{10}^{0} = 2(1-e^{-.2})$$

$${}_{10}q_{75}^{(\tau)} = 1 - {}_{10}p_{75}^{(\tau)} = 1 - {}_{10}p_{75}^{\prime(1)} \cdot {}_{10}p_{75}^{\prime(2)} = 1 - \frac{15}{25} \cdot e^{-.2} = 1 - .6e^{-.2}$$

$$\therefore {}_{10}q_{75}^{(2)} = (1-.6e^{-.2}) - (2-2e^{-.2}) = 1.4e^{-.2} - 1 = 0.146\cdots$$

4. For a double decrement model, you are given:

(i) $\mu_{x+t}^{(1)} = 2t$

(ii) 50% of decrement 2 occurs at time $t = 0.2$, and the rest occurs at time $t = 0.8$

(iii) $q_x^{(2)} = 0.3$

Determine $q_x'^{(2)}$.

$$q_x^{(2)} = {}_{.2}p_x'^{(1)} \cdot (.5 \cdot q_x'^{(2)}) + {}_{.8}p_x'^{(1)} (.5 \cdot q_x'^{(2)})$$

$$\therefore \quad q_x'^{(2)} = \frac{2 \cdot q_x^{(2)}}{{}_{.2}p_x'^{(1)} + {}_{.8}p_x'^{(1)}} = \frac{.6}{{}_{.2}p_x'^{(1)} + {}_{.8}p_x'^{(1)}}$$

$${}_n p_x'^{(1)} = \cancel{{}_0 8}\, e^{-\int_0^n \mu_{x+t}^{(1)} dt} = e^{-\int_0^n 2t\, dt} = e^{-n^2}$$

$$\Longrightarrow \quad {}_{.2}p_x'^{(1)} = e^{-(.2)^2} = e^{-.04}$$

$${}_{.8}p_x'^{(1)} = e^{-(.8)^2} = e^{-.64}$$

$$\therefore \quad q_x'^{(2)} = \frac{.6}{e^{-.04} + e^{-.64}} = 0.4032\ldots$$

5. For a triple decrement model, you are given:

(i) $q_x^{(1)} = 0.1$ and decrements 1 is uniformly distributed in the associated single decrement table

(ii) $q_x^{(2)} = 0.2$ and decrements 2 is uniformly distributed in the associated single decrement table

(iii) $q_x^{(3)} = 0.3$ and decrement 3 is a beginning of year decrement $\Rightarrow q_x^{(3)} = q_x^{\prime(3)} = .3$

Determine ${}_{0.5}q_x^{\prime(1)}$.

$$(i) \Rightarrow {}_{.5}q_x^{\prime(1)} = .5 \cdot q_x^{\prime(1)} \quad (\text{Need } q_x^{\prime(1)})$$

$$0.1 = q_x^{(1)} = \int_0^1 \underbrace{{}_tp_x^{\prime(3)}}_{= .7 \text{ since (3) is a BOY decrement}} \cdot \underbrace{{}_tp_x^{\prime(2)}}_{= 1 - t\cdot q_x^{\prime(2)} \text{ since (2) is SUDD}} \cdot \underbrace{{}_tp_x^{\prime(1)} \cdot \mu_{x+t}^{(1)}}_{= q_x^{\prime(1)} \text{ since (1) is SUDD}} \, dt$$

$$\therefore .1 = .7 \cdot q_x^{\prime(1)} \cdot \int_0^1 (1 - t\cdot q_x^{\prime(2)})\,dt = .7 \cdot q_x^{\prime(1)} \left(1 - \frac{q_x^{\prime(2)}}{2}\right)$$

Rewriting, we have $\quad .1 = .7 \cdot q_x^{\prime(1)} \cdot \left(1 - \frac{q_x^{\prime(2)}}{2}\right)$

Likewise $\quad .2 = .7 \cdot q_x^{\prime(2)} \cdot \left(1 - \frac{q_x^{\prime(1)}}{2}\right)$

In order to simplify notation, let $q_x^{\prime(1)} = a$ $(0<a<1)$ and $q_x^{\prime(2)} = b$ $(0<b<1)$

$$\therefore \begin{cases} 1 = 7a(1 - .5b) \\ 2 = 7b(1 - .5a) \Rightarrow b = \dfrac{2}{7(1-.5a)} \end{cases}$$

$$\therefore 1 = 7a\left(1 - .5\frac{2}{7 - 3.5a}\right) = 7a \cdot \left(\frac{6 - 3.5a}{7 - 3.5a}\right)$$

$$\Rightarrow 7 - 3.5a = 7a(6 - 3.5a) \Rightarrow 24.5a^2 - 45.5a + 7 = 0$$

$$\Rightarrow a = \frac{45.5 \pm \sqrt{(-45.5)^2 - 4(24.5)(7)}}{2(24.5)} = 0.169\ldots$$

$$\therefore {}_{.5}q_x^{\prime(1)} = .5 \cdot q_x^{\prime(1)} = .5 \cdot a = 0.0846\ldots$$