MAP 4175 / 5177
Test 8

Name: KEY

Date: October 24, 2018

Each problem is worth 10 points. Show all work for full credit, and use correct notation. Simplify answers completely. See other side for additional problems.

1. Given $\mu_x = 0.02$, $\mu_y = 0.03$, and $\mu_{xy} = 0.04$, determine ${}_{10}q_{\overline{xy}}$

For a constant force, ${}_nP = e^{-\mu \cdot n}$

$$\therefore \quad {}_{10}P_x = e^{-0.2}, \quad {}_{10}P_y = e^{-0.3}, \quad {}_{10}P_{xy} = e^{-0.4}$$

$$\Rightarrow {}_{10}P_{\overline{xy}} = e^{-0.2} + e^{-0.3} - e^{-0.4} = 0.889\ldots$$

$$\therefore {}_{10}q_{\overline{xy}} = 0.110\ldots$$

2. Determine the value of $T_{\overline{xy}}$ if $T_x + T_y = 26.3$ and $T_x T_y = 172.5$.

Let $a = T_x$ & $b = T_y$

$$a + b = 26.3, \quad ab = 172.5$$

$$\Rightarrow a \cdot (26.3 - a) = 172.5$$

$$\Rightarrow a^2 - 26.3a + 172.5 = 0$$

$$\therefore a = 12.5 \text{ or } 13.8$$

$$a = T_x = 12.5 \text{ or } 13.8, \quad T_y = \text{the other}$$

$$\Rightarrow T_{\overline{xy}} = \max(T_x, T_y) = 13.8$$

3. Given $\overset{o}{e}_{30:\overline{10}|} = 9$, $\overset{o}{e}_{40:\overline{10}|} = 8.75$, and $\overset{o}{e}_{30:\overline{20}|} = 16$, determine ${}_{10}q_{30}$.

$$\overset{o}{e}_{30:\overline{20}|} = \overset{o}{e}_{30:\overline{10}|} + {}_{10}p_{30} \cdot \overset{o}{e}_{40:\overline{10}|}$$

$$16 = 9 + {}_{10}p_{30} \cdot (8.75)$$

$$\Rightarrow {}_{10}p_{30} = 0.8$$

$$\therefore {}_{10}q_{30} = 0.2$$

4. Given $q_{60} = 0.04$ and $e_{61} = 17.5$, determine e_{60}.

$$e_{60} = p_{60} \cdot (1 + e_{61})$$

$$= 0.96 \cdot (1 + 17.5)$$

$$\therefore e_{60} = 17.76$$

5. Given ${}_tp_{xy} = (1.05)^{-t}$, determine $e_{xy:\overline{10}|}$

$$e_{xy:\overline{10}|} = p_{xy} + {}_2p_{xy} + \cdots + {}_{10}p_{xy}$$

$$= (1.05)^{-1} + (1.05)^{-2} + \cdots + (1.05)^{-10}$$

$$= v + v^2 + \cdots + v^{10} \qquad \left(v = \frac{1}{1.05}\right)$$

$$\therefore e_{xy:\overline{10}|} = a_{\overline{10}|.05} = 7.721\cdots$$